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Sub-Planckian Black Holes of the Variable Energy Density, Discretization of Spacetime and Modified Friedmann Equations

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Abstract. A model of a three-dimensional dynamic quantum vacuum defined by a fundamental variable energy density, where microphysics and macrophysics emerge from elementary sub-Planckian black holes with a size of order of a generalized Compton wavelength that unifies the standard Compton wavelength and Schwarzschild radius, is developed. The granular features of the texture of the sub-Planckian black holes at their generalized Compton wavelength are explored. Modified Friedmann equations associated with the sub-Planckian black holes of the variable energy density are introduced and cosmological consequences are derived in the picture of a cyclic universe. This approach avoids the presence of singularities and allows an unifying treatment of the different phases of the evolution of the universe.

KEY WORDS: variable energy density, generalized Compton wavelength, sub-Planckian black holes, modified Friedmann equations, cyclic universe.

1 Introduction

Despite the extraordinary results that the scientific revolutions of the XX century have brought us, the description of the universe provided by the two fundamental theories we have, i.e. quantum theory and general relativity, is someway fragmented and incomplete and needs to be adjusted. The search of an adequate and fruitful unification of general relativity and quantum mechanics is perhaps the most important challenge of contemporary theoretical physics. In this regard, an interesting line of research is recently receiving attention where a unifying treatment of the Compton scale of microphysics of elementary particle and the Schwarzschild scale of black holes can be obtained in the context of a generalized uncertainty principle, that posits the existence of a minimal length and a breakdown of the usual Heisenberg uncertainty principle at the Planck scale [1–6]. Different forms of generalized uncertainty relations have been developed, that lead to the existence of a minimum measurable length and have the merit to introduce suggestive perspectives to treat the quantization of gravitational fields as well as issues of quantum cosmology, black holes and the quantum vacuum [7–21].

In this paper, we want to explore the cosmological consequences of a peculiar form of generalized uncertainty relations in a model of three-dimensional quantum vacuum, developed by the author of this paper in a series of works [22–30], where the observable events of the physical world, both in the macroscopic domain and in the microscopic regime, are derived from a fundamental quantum foam, i.e. a three-dimensional (3D) dynamic quantum vacuum (DQV) described by a variable energy density. An important merit of this model is that ordinary matter, dark matter and dark energy can be seen as emergent structures, forms of collective organizations, which come into existence as the upper manifestations of specific states of the DQV, which correspond to specific fluctuations of the underlying variable quantum vacuum energy density.

The ultimate geometry of the 3D DQV is described by a peculiar specific form of generalized uncertainty relations, which are valid at the Planck scale [30]:

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1+\beta l_p^2 \frac{\Delta\rho_{qvE}^2 V^2}{\hbar^2 c^2}\right) . \tag{1}$$

In equation (1) $\hbar$ is Planck's reduced constant, $\Delta\rho_{qvE}$ is the variable energy density of the vacuum corresponding to the appearance of a material particle of mass m in a volume V, defined as

$$\Delta\rho_{qvE} \equiv \rho_{pE} - \rho_{qvE} = \frac{mc^2}{V} , \tag{2}$$

where

$$\rho_{PE} = \frac{m_p c^2}{l_p^3} \tag{3}$$

is Planck's energy density (m_P being Planck's mass, c the light speed and l_p Planck's length) intended as the ground state of the 3D DQV, β is a fluctuating dimensionless parameter which expresses the fact that here space-time fluctuations fix the minimal length scale only on average and the term $\frac{\hbar}{2}\beta l_p^2 \frac{\Delta\rho_{qvE}^2 V^2}{\hbar^2 c^2}$ measures the degree of violations of the Heisenberg uncertainty relations at scales that approach the Planck scale.

By starting from the generalized uncertainty relations (1), the variable energy density of the 3D DQV has the fundamental property of leading to the following fundamental scale where Compton wavelength and Schwarzschild radius are unified:

$$R'_C = R'_S = \sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE} V}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE} V}{\hbar c}\right)^2} , \tag{4}$$

which can be defined as generalized Compton wavelength [30]. On the basis of equation (4), the standard Compton wavelength and Schwarzschild radius can be seen as special cases, in opportune regimes, of the generalized Compton wavelength, as a consequence of corresponding underlying energy density fluctuations of the 3D DQV. Because of the role of the generalized Compton wavelength as the fundamental property of nature that determines the emergence of elementary particles or black holes of macrophysics whether of the cases, in the model of the 3D DQV, at a fundamental level one can say that the ultimate origin of microphysics and macrophysics lies in a texture of elementary objects of the Planck scale, which we name sub-Planckian black holes with a size of order of their generalized Compton wavelength, which are generated by the geometrodynamic properties of the variable quantum vacuum energy density ultimately associated to the foam of the virtual particles of the vacuum [30].

In this paper, our aim is to show how the texture of sub-Planckian black holes of the quantum vacuum energy density at the generalized Compton wavelength, besides determining a discretization of spacetime, leads to suggestive consequences as regards cosmology, by inducing important modifications in the Friedmann equations that introduce a new key of reading of the paradigm of the cyclic universe. The structure of the paper is the following. In Section 2 we explore the granular and holographic features of the texture of the sub-Planckian black holes of the variable energy density at the generalized Compton wavelength. In Section 3 we show how, by starting from the dependence of entropy of a black hole with the texture of sub-Planckian black holes of the variable energy density, modified Friedmann equations may be derived. In Section 4, we explore the cosmological perspectives of the modified Friedmann equations of the 3D DQV in the picture of the cyclic universe. Finally, in Section 5 we summarize the results of the paper.

2 About the Discretization of the Texture of Sub-Planckian Black Holes of the Variable Energy Density

Sub-Planckian black holes of the variable energy density of the 3D DQV imply the existence of a minimal length, and thus a quantum nature of the fundamental background, providing a new way to re-read the discretization of space.

If loop quantum gravity suggests that in the quantum-gravity domain physical space is composed by elementary grains, a net of intersecting loops, having approximately the Planck size [31–33], thus determining a geometry described by a discrete quantized 3D metric, in a similar way, in the model of the 3D DQV where the elementary fluctuations are described by sub-Planckian black holes of the variable energy density, one deals with a granular structure of space at the fundamental scale represented by the generalized Compton wavelength. In affinity with loop quantum gravity, the discretization of quantum geometry at the Planck scale can be here considered as a genuine property of space, but, while in loop quantum gravity it is independent of the strength of the actual gravitational field at any given location, in our model it is ultimately derived from the actual variable energy density of the 3D DQV.

The generalized uncertainty relation (1), in the light of the existence of the fundamental scale of the generalized Compton wavelength (4), implies that the geometry of the 3D DQV has a granular structure where the generalized Compton wavelength represents the fundamental unit of measure of length. In other words, one can say that the geometry of the 3D DQV is defined by cells whose size is directly fixed by the behaviour of the quantum vacuum energy density through the generalized Compton wavelength, i.e.

$$\Delta x \gtrsim \Delta x_{\min} = \sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2}\,. \tag{5}$$

The physical meaning of equation (5) is that the energy fluctuations of the 3D DQV are characterized by a granularity in the sense that correspond to elementary cells whose size is directly determined by the quantity $\sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2}$. In other words, as a consequence of the corresponding behavior of the quantum vacuum energy density fluctuations, the 3D DQV background is characterized by a minimum measurable accessible length $\sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2}$, i.e., at the most fundamental level, is given by a texture of elementary cells of dimensions fixed by the generalized Compton wavelength $\sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2}$.

On the other hand, in the model of the 3D DQV, one can introduce a generalized time-energy uncertainty relation given by expression:

$$\Delta E \Delta t \geq \frac{\hbar}{2}\left(1 + \beta l_p^2 \frac{\Delta\rho_{qvE}^2 V^2}{\hbar^2 c^2}\right)\,. \tag{6}$$

The generalized time-energy relation (6) leads to derive a minimum measurable time scale determined by the texture of the sub-Planckian black holes of the variable energy density of the 3D DQV, of the form

$$\Delta t \gtrsim \Delta t_{\min} = \frac{1}{c}\sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2}\,. \tag{7}$$

The quantity (7) can be defined as the lifetime of vacuum fluctuations with localization associated to the minimum measurable distance (5). On the basis of equations (5) and (7), if the wavelength of the virtual particles of the vacuum satisfies relation $\lambda \gtrsim \Delta x_{\min}$ and the duration of the process of appearance of the virtual particles of the vacuum satisfies $\Delta t \gtrsim \Delta t_{\min}$, then the corresponding quantum fluctuations give rise to actualizations beyond the cube of side $\Delta x_{\min}$

centered on the point where the fluctuation occurs with a duration beyond the minimum measurable time scale $\Delta t_{\min}$.

On the basis of (and compatibly with) the results of loop quantum gravity [34], a classical macroscopic 3D gravitational field which determines a 3D metric $g_{\mu\nu}$ can be considered as an emergent fact from the fundamental 3D DQV defined by sub-Planckian black holes of the variable energy density and, if we consider a region R of area S with a size much larger than $l \gg l_P$ and slowly varying at this scale, the size of this region can be expressed in terms of the generalized Compton wavelength. The volume and the area of each physical region derive from the granular features of the 3D DQV, i.e. here one can consider a volume operator $V(R)$ and an area operator $A(S))$ that, in analogy with loop quantum gravity, have a quantum spectrum with eigenvalues equal to the volume of R (and of the area of S) that are determined by the metric $g_{\mu\nu}$ ultimately associated with the variable energy density and therefore are a function of the generalized Compton wavelength.

In affinity with the spin network states of the loop quantum gravity, in the 3D DQV model, in order to define physical states on a 3D manifold with coordinates $\vec{x}$ that approximate the flat 3D metric $g^{(0)}_{\mu\nu}(\vec{x}) = \eta_{\mu\nu}$, one can build a spatially uniform state $|S_{\mu_0}\rangle$ constituted by a set of sub-Planckian black holes of the variable energy density of coordinate density $\rho = \mu_0^{-2}$ where the average distance μ_0 from each other, i.e. the "lattice spacing" μ_0, can be associated to the generalized Compton wavelength. Therefore, if μ is the density of the sub-Planckian black holes of the variable energy density, by increasing the coordinate density of the loops, one obtains a correspondent increasing of the area

$$A(S)|S_\mu\rangle \approx \frac{\mu_0^2}{\mu^2}\left(A[g^{(0)},S] + O(l/l_P)\right)|S\rangle\,. \tag{8}$$

Since the action of the area operator is

$$\frac{\mu_0^2}{\mu^2}A[g^{(0)},S] = \left(A\left[\frac{\mu_0}{\mu}g^{(0)},S\right] + O(l/l_P)\right) = A\left[g^{(\mu)},S\right]\,, \tag{9}$$

the state of the vacuum characterized by an increased loop density approximates the metric

$$g^{(\mu)}_{\mu\nu} = \frac{\mu_0^2}{\mu^2}\eta_{\mu\nu} \tag{10}$$

that, taking account of the role of the generalized Compton wavelength as fundamental scale of nature, becomes

$$g^{(\mu)}_{\mu\nu} = \frac{1}{\mu^2}\Big[\Big(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\Big)^2 + \Big(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\Big)^2\Big]\eta_{\mu\nu}\,. \tag{11}$$

On the basis of equation (11), we can say that, at a fundamental level, the 3D DQV defined by the sub-Planckian black holes of the variable energy density, has granular features associated with the fundamental scale represented by the generalized Compton wavelength.

Moreover, the granular nature of the geometry of the 3D DQV implies a corresponding quantization of the area. In this regard, compatibly with the results of loop quantum gravity, one can write the smallest possible value of the area of a region of the 3D DQV as:

$$A_{\min} = 4\pi\sqrt{3}\gamma \left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V} \right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar c} \right)^2 \right], \tag{12}$$

where γ is the usual Immirzi parameter of order 1. In this picture, the quantity $b_0 = A_{\min}/(8\pi)$ quantifies the deviation from the classical theory. As a consequence, in the geometry of the 3D DQV defined by sub-Planckian black holes of the variable energy density, the distance associated with the outer event horizon can be expressed as

$$R'_C = R'_S = \sqrt{\left(\frac{\hbar c}{\Delta\rho_{qvE}V} \right)^2 + \left(b_0 \frac{c^4}{G\Delta\rho_{qvE}V} \right)^2}\,. \tag{13}$$

In summary, the physical meaning of the approach based on equations (8)–(13) is that, in the texture of the sub-Planckian black holes of the variable energy density of the 3D DQV, a smooth geometry cannot be approximated at a physical scale lower than the generalized Compton wavelength at the Planck scale.

Another significant feature of the texture of the states of the fundamental quantum geometry of the 3D DQV defined by the sub-Planckian black holes of the variable energy density lies in its holographic nature. By invoking fruitful considerations proposed by Ng in the context of his holographic model of fundamental spacetime [35–38], we can assume that the quantum fluctuations of the 3D DQV corresponding to the behaviour of the sub-Planckian black holes of the variable energy density manifest themselves in the form of uncertainties in the geometry of this background and, thus, that the measurement of the radius l of a spherical volume is characterized by an average minimum uncertainty of the form

$$\delta l = l^{1/3} \left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V} \right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar c} \right)^2 \right]^{1/3}\,. \tag{14}$$

On the basis of equation (14), since on the average each cell corresponding to a sub-Planckian black hole of the variable energy density occupies a spatial volume of $l \left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V} \right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar c} \right)^2 \right]$, a spatial region of size l can contain a maximum number of cells

$$\frac{l^3}{l \left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V} \right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar c} \right)^2 \right]} = \frac{l^2}{\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V} \right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar c} \right)^2 \right]}\,,$$

corresponding to bits of information. The existence of a maximum number of bits of information contained in a spatial region is in agreement with the holographic principle [39–44] which implies that, although the world around us appears to have three spatial dimensions, its contents can actually be encoded on a two-dimensional surface, like a hologram. As a consequence, a fundamental feature of the background geometry of the 3D DQV defined by sub-Planckian black holes of the variable energy density lies in its holographic and non-local features at the generalized Compton wavelength, in the sense that the physical degrees of freedom must be considered as infinitely correlated, with the result that the spacetime localization of an event may lose its invariant significance, compatibly with Ng's model.

Moreover, in our approach, we consider that the appearance of an elementary particle corresponds to the switching of opportune cells of the 3D DQV and that this process can be associated with a specific duration τ_0, which therefore can be defined as the minimum proper time corresponding to the switching of a cell, which generates the "bare" state of a particle (and thus determines, as a derived fact, the time interval between two successive localizations of the same particle). As a consequence, this actualization event will be characterized by an intrinsic positional uncertainty of the form

$$\delta l \geq \left(\frac{2\pi^2}{3}\right)^{1/3} c\tau_0 \, . \tag{15}$$

In the light of equation (15), the holographic features of the 3D DQV constituted by sub-Planckian black holes of the variable energy density, imply that a sphere that does not collapse into a black hole physically corresponds to the action of a maximum energy density given by relation

$$\rho = \frac{9}{16\pi^2} \frac{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2}{(c\tau_0)^6} \, . \tag{16}$$

Now, the intrinsic positional uncertainty (15) associated with the quantum fluctuations generates a corresponding spatial length, which we denote l_0, expressing the minimum spatial length, where the switching of opportune cells corresponding to sub-Planckian black holes of the variable energy density of the 3D DQV can give origin to the appearance/actualization of the "bare state" of a particle, such as a fermion of the Standard Model. In order to define this peculiar spatial length characterizing the ultimate texture of sub-Planckian black holes of the variable energy density of the 3D DQV, by taking account of Ng's results, we assume that the spatial volume occupied on the average by each cell is $l_0\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2\right]$, and thus that a spatial region of size l_0

contains a number of cells given by $l_0^2 \Big/ \left[\left(\frac{\beta \hbar c}{\Delta\rho_{qvE} V} \right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE} V}{\hbar c} \right)^2 \right]$. We can therefore define l_0 through the following equation:

$$(\delta l)^3 = l_0 \left[\left(\frac{\beta \hbar c}{\Delta\rho_{qvE} V} \right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE} V}{\hbar c} \right)^2 \right] \tag{17}$$

which yields

$$l_0 = \frac{2\pi^2}{3} \frac{(c\tau_0)^3}{\left(\frac{\beta \hbar c}{\Delta\rho_{qvE} V} \right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE} V}{\hbar c} \right)^2} \,. \tag{18}$$

Equation (18) expresses, in the 3D DQV background, the minimum size of each spatial length which is ultimately determined by the intrinsic positional uncertainty (15) characterizing the lattice of sub-Planckian black holes of the variable energy density, which gives rise to the appearance of the "bare" state of a particle. Taking account that the quantity (18) represents the minimum length which is physically accessible and has physical meaning in the sense that corresponds to the actualization of the bare state of an elementary particle, one can say that the value (18) is the minimum value the generalized Compton wavelength can assume. Therefore, by equating equation (18) and equation (4), one finds

$$\sqrt{\left(\frac{\beta \hbar c}{\Delta\rho_{qvE} V} \right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE} V}{\hbar c} \right)^2} = \frac{2\pi^2}{3} \frac{(c\tau_0)^3}{\left(\frac{\beta \hbar c}{\Delta\rho_{qvE} V} \right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE} V}{\hbar c} \right)^2} \tag{19}$$

that yields

$$\sqrt{\left(\frac{\beta \hbar c}{\Delta\rho_{qvE} V} \right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE} V}{\hbar c} \right)^2} = \left(\frac{2\pi^2}{3} \right)^{1/3} c\tau_0 \,, \tag{20}$$

namely

$$\tau_0 = \frac{\sqrt{\left(\frac{\beta \hbar c}{\Delta\rho_{qvE} V} \right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE} V}{\hbar c} \right)^2}}{\left(\frac{2\pi^2}{3} \right)^{1/3} c} \,. \tag{21}$$

On the basis of equation (21), we can say that also the minimum duration of the process of the switching of the cells of the 3D DQV, that generates the appearance of an elementary particle in its "bare" state, depends – and is directly fixed by – the variable quantum vacuum energy density through the generalized Compton wavelength.

In summary, we can say that, in the model of the sub-Planckian black holes of the variable energy density, the discretization of spacetime, its holographic features as well as the existence of a minimum length corresponding to the actualization of an elementary particle in its "bare" state, are directly connected with the generalized Compton wavelength intended as a fundamental scale of nature.

3 The Modified Friedmann Equations in the Texture of the Sub-Planckian Black Holes of the Variable Energy Density of the Three-Dimensional Quantum Vacuum

The motion of a test particle in the holographic surface of the 3D DQV background generates a modification of the entropy of the surface by fundamental units depending on the discrete spectrum of the area of the surface. Therefore, in order to explore the cosmological implications of the discretization of spacetime determined by the generalized uncertainty relations (1) and (6), let us start by analysing the impact on the Bekenstein-Hawking entropy of a black hole. In this regard, we consider that, for a black hole that absorbs or emits a quantum particle of energy E and size R_0, its area A is characterized by a modification of amount:

$$\Delta A \geq 8\pi \Delta x_{\min}^2 E R_0 , \tag{22}$$

where $\Delta x_{\min}$ is given by relation (5). In this way, the generalized uncertainty relation (1), and the consequent existence of the minimum measurable scale (5) in the lattice of sub-Planckian black holes of the variable energy density of the 3D DQV, lead to the following result as regards the minimum change of the area that is physically possible:

$$\Delta A_{\min} \gtrsim \frac{8\pi\Delta x^2}{2\beta}\left[1-\sqrt{1-\frac{1}{\Delta x^2}\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2\right]}\right]. \tag{23}$$

On the basis of equation (23), as a consequence of the absorption or emission of a quantum particle, the black hole area overcomes a modification induced by the quantum fluctuations of the vacuum associated with the texture of the sub-Planckian black holes of the variable quantum vacuum energy density. Here, by following [45–48], one can assume that the value of Δx is the inverse of surface gravity and thus can be associated to the Schwarzschild radius r_s namely $\Delta x = 2r_s$, finding the following identity:

$$\Delta x^2 = \frac{A}{\pi} . \tag{24}$$

Substituting relation (24) inside equation (23), one obtains

$$\Delta A_{\min} \gtrsim \frac{8A}{2\beta}\left[1-\sqrt{1-\frac{1}{A}\pi\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2\right]}\right] \tag{25}$$

and therefore the modification of the area of the black hole becomes

$$\Delta A_{\min} \gtrsim \frac{8A}{2\beta}\lambda\left[1-\sqrt{1-\frac{1}{A}\pi\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2\right]}\right]. \quad (26)$$

In equation (26), the parameter λ can be determined by the Bekenstein-Hawking relation. Taking account that, on the basis of the information theory [49], a minimal increase of entropy does not depend on the horizon's area, equation (26) leads to relation

$$\frac{dS}{dA} = \frac{\Delta S_{\min}}{\Delta A_{\min}} = \frac{b}{\frac{8A}{2\beta}\lambda\left[1-\sqrt{1-\frac{1}{A}\pi\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2\right]}\right]}, \quad (27)$$

where, on the basis of the Bekenstein-Hawking formula, $b/\lambda = 2\pi$ and therefore we obtain

$$\frac{dS}{dA} = \frac{\Delta S_{\min}}{\Delta A_{\min}} = \frac{\pi}{\frac{2A}{\beta}\left[1-\sqrt{1-\frac{1}{A}\pi\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2\right]}\right]}. \quad (28)$$

By integrating equation (28), the following expression of the entropy is obtained:

$$S = \frac{1}{8\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2\right]} \times\left\{A+\sqrt{A^2-\pi\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2\right]A} - \pi\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2\right] \times\ln\left(A+\sqrt{A^2-\pi\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2\right]A} - \pi\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2\right]\right)\right\} + S_0, \quad (29)$$

where S_0 is an integration constant. The physical meaning of equation (29) is that the Bekenstein-Hawking entropy of a black hole is modified as a consequence of the minimum length scale determined by the generalized uncertainty relation (1) expressing the ultimate geometry of the fluctuations of the vacuum associated with the texture of the sub-Planckian black holes of the variable quantum vacuum energy density. In other words, the activity of sub-Planckian black holes of the variable energy density at the generalized Compton wavelength induces changes into the Bekenstein-Hawking entropy of a black hole which turn out to be relevant in the quantum-gravity regime.

Moreover, we remark here that, on the basis of the modified entropy-area relation (28), we can provide an entropic treatment of gravity force and interpret gravity as an emergent phenomenon. In this regard, in affinity with Verlinde's and Padmanaban's proposals [50, 51], we can consider the thermodynamic equation of state

$$F = T\frac{\Delta S}{\Delta x} \tag{30}$$

where ΔS is one fundamental unit of entropy, Δx satisfies equation (5) and thus corresponds to a certain number N of elementary cells of the 3D DQV, the entropy gradient is directed radially from the outside of the surface to inside and T is the temperature of the surface that is related to the total energy of the surface by

$$E_{\text{tot}} = \frac{1}{2}Nk_BT\,, \tag{31}$$

where k_B is the Boltzmann constant. From equations (30) and (31), it is possible to derive a modified Newton's law of gravitation of the form

$$F = -G\frac{Mm}{R^2}\left[1+4\frac{dS}{dA}\right]_{A=4\pi R^2}, \tag{32}$$

where m is the mass of the test particle absorbed or emitted by the black hole, M is the relativistic rest mass of the source associated with the surface of the region of the 3D DQV into consideration, R is the radius of the horizon. Moreover, by invoking a T-duality and using the insights made in [52, 53], one can consider the following modified version of the gravitational potential in the texture of the sub-Planckian black holes of the variable energy density at their generalized Compton wavelength:

$$\phi(r) = -G\left.\frac{Mm}{\sqrt{r^2+\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2\right]}}\right|_{r=R}, \tag{33}$$

which leads to express the modified Newton's law of gravitation (32) as

$$F = -G\frac{Mm}{R^2}\left[r^2+\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2+\left(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2\right]\right]^{-3/2}. \tag{34}$$

On the basis of equations (32) and (34), one can say that the texture of sub-Planckian black holes of the variable energy density generates modifications into Newton's law of gravitation, that turn out to be important in the quantum-gravity regime, compatibly with the results obtained in [54].

Now, on the basis of the assumption that any modification of the area-law coming from the physics of black holes would imply a similar modification to the area-law that one should use to study the apparent horizon of the universe, we will show how the modified entropy (29) induces a deformation of the Friedmann equations, that rule the dynamics of the geometry of the universe. In this regard, by following [55], we start by expressing the link between entropy and area through the following relations

$$S = \frac{f\left(A\right)}{4G}\,, \tag{35}$$

$$\frac{dS}{dA} = \frac{f'\left(A\right)}{4G}\,, \tag{36}$$

where A is the area of the apparent horizon (intended as a marginally trapped region characterized by a vanishing expansion, that always exists in a FLRW universe), $f\left(A\right)$ is any arbitrary smooth function of the area of the apparent horizon and $f'\left(A\right) = \frac{df\left(A\right)}{dA}$. Thus, by using the first law of thermodynamics, one finds

$$TdS = -\frac{f'\left(A\right)}{G}\left(1 - \frac{\dot{r}_A}{2Hr_A}\right)dr_A\,, \tag{37}$$

where $r_A = c\big/\sqrt{H^2 + kc^2/a^2}$ is the radius of the apparent horizon, a is the Friedmann-Robertson-Walker scale factor that parametrizes the expansion/contraction of the universe during the evolution, $H = \dot{a}/a$ is the Hubble parameter, k is the spatial curvature. By inserting (36) inside equation

$$TdS = 4\pi\left(\rho + p\right)\left(\frac{\dot{r}_A}{2Hr_A} - 1\right)Hr_A^3 dt\,, \tag{38}$$

where ρ is the energy density and p is the pressure matter in the universe, one obtains equation

$$\left(\dot{H} - \frac{kc^2}{a^2}\right)f^{'}\left(A\right) = -4\pi G\left(\rho + p\right). \tag{39}$$

Then, by using the continuity equation

$$\dot{\rho} + 3H\left(\rho + p\right) = 0 \tag{40}$$

and relation

$$dr_A = -Hr_A^3\left(\dot{H} - \frac{kc^2}{a^2}\right)dt\,, \tag{41}$$

equation (39) becomes

$$f^{'}\left(A\right)\frac{dr_{A}}{r_{A}^{3}}=-\frac{4\pi G}{3}d\rho \tag{42}$$

and, by integrating (42), one obtains the following modified Friedmann equations:

$$\frac{2G}{3}\rho=-\int f'\left(A\right)\frac{dA}{A^{2}}\,. \tag{43}$$

The physical meaning of equations (43) is that a different modified Friedmann equation corresponds to each different apparent horizon associated to each different area-law for the entropy, thus implying the perspective of a multiverse scenario.

Now, compatibly with the algebra formulated in [55], one has the possibility to express the modified Friedmann equations (43) in the vacuum of sub-Planckian black holes of the variable quantum vacuum energy density ruled by the generalized uncertainty relations (1). In this regard, by substituting equation (28) inside (43), one arrives at relations

$$\frac{\left(\dot{H}-\frac{kc^{2}}{a^{2}}\right)\frac{1}{2}\pi\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^{2}+\left(\beta l_{p}^{2}\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^{2}\right]}{A-\sqrt{A^{2}-\pi\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^{2}+\left(\beta l_{p}^{2}\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^{2}\right]A}}=-4\pi G(\rho+p) \tag{44}$$

and

$$\frac{8\pi G}{3}\rho=2\pi\left\{\frac{1}{A}-\frac{2\left(A^{2}-\pi\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^{2}+\left(\beta l_{p}^{2}\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^{2}\right]A\right)^{3/2}}{3\pi\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^{2}+\left(\beta l_{p}^{2}\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^{2}\right]A^{3}}\right\}+C, \tag{45}$$

where the integration constant C, taking account that, as a consequence of the expansion of the universe in the post big bang era, the area $A=\frac{4\pi c^{2}}{H^{2}+kc^{2}/a^{2}}$ of the apparent horizon goes to infinity in correspondence to a vacuum energy density equal to the cosmological constant Λ, is given by expression

$$C=\frac{8\pi G}{3}\left(\Lambda+\frac{1}{2G\pi\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^{2}+\left(\beta l_{p}^{2}\frac{\Delta\rho_{qvE}V}{\hbar c}\right)^{2}\right]}\right). \tag{46}$$

In this way, one finally arrives at the following modified Friedmann equations in the vacuum of sub-Planckian black holes of the variable quantum vacuum energy density characterized by a discretization of spacetime at Planck scale:

$$\frac{8\pi G}{3}(\rho-\Lambda)=\frac{1}{2}\Big(H^2+\frac{kc^2}{a^2}\Big)+\frac{2}{3\Big[\Big(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\Big)^2+\Big(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\Big)^2\Big]}$$
$$\times\left[1-\left(1-\frac{\Big(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\Big)^2+\Big(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\Big)^2}{2}\Big(H^2+\frac{kc^2}{a^2}\Big)\right)^{3/2}\right], \quad (47)$$

$$-4\pi G(\rho+p)=\Big(\dot{H}-\frac{kc^2}{a^2}\Big)\frac{1}{4}\left[\Big(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\Big)^2+\Big(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\Big)^2\right]$$
$$\times\frac{\Big(H^2+\frac{kc^2}{a^2}\Big)}{1-\left(1-\frac{1}{2}\Big[\Big(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\Big)^2+\Big(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\Big)^2\Big]\Big(H^2+\frac{kc^2}{a^2}\Big)\right)^{1/2}}. \quad (48)$$

We observe here that, if the second term on the right-hand side of equation (47) tends to zero, then equation (47) reduces to the standard dynamical Friedmann equation with cosmological constant

$$\frac{8\pi G}{3}(\rho-\Lambda)=\frac{1}{2}\left(H^2+\frac{kc^2}{a^2}\right). \quad (49)$$

In other words, we can say that the quantity

$$\frac{2}{3\Big[\Big(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\Big)^2+\Big(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\Big)^2\Big]}$$
$$\times\left[1-\left(1-\frac{\Big(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\Big)^2+\Big(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\Big)^2}{2}\Big(H^2+\frac{kc^2}{a^2}\Big)\right)^{3/2}\right],$$

which is connected with the activity of sub-Planckian black holes at their generalized Compton wavelength, evaluates the departure degree of Friedmann equations from the classical behavior predicted by general relativity.

It is interesting to observe that the modified Friedmann equations (47) and (48), determined by the generalized uncertainty relations (1) describing the geometry of the vacuum of the sub-Planckian black holes of the variable quantum vacuum energy density, imply the existence of a maximum energy density with a value around Planck density for any equation of state and all values of k. In fact, on the basis of equation (47), the density is real if the following constraint regarding the area of the apparent horizon is satisfied:

$$H^2 + \frac{kc^2}{a^2} \leq \frac{2}{\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2\right]} \,. \tag{50}$$

The constraint (50) implies that the scale factor must have a maximum and the Hubble parameter a minimum, thus leading to a maximum energy density

$$\rho_{\max} = \Lambda + \frac{5}{4G\left[\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2\right]} \approx \rho_P \,, \tag{51}$$

where ρ_P is Planck's density. As a consequence, by assuming a general continuous pressure $p(\rho,\, a)$, if the scale factor has a maximum value and the Hubble parameter is bounded, it follows that all curvature invariants of the Friedmann-Robertson-Walker metric turn out to be finite and universe is prevented from reaching a singularity and has therefore a general non-singular evolution. The discretization of the background expressed by the existence of the minimal length (5) has therefore the merit to remove the singularities present in general relativity and, just like the standard uncertainty principle avoids the collapse of the hydrogen atom, in the same way the ultimate geometrodynamics characterizing the texture of sub-Planckian black holes of the variable energy density of the 3D DQV induces modifications in the entropy of the black holes that prevent the catastrophic nature of the Hawking radiation, thus introducing new perspectives of solution of the black hole information and firewall paradoxes.

4 Cosmological Implications Towards a Cyclic Universe

The modified Friedmann equations (47) and (48) ruling the dynamics of the geometry of the background of the sub-Planckian black holes of the variable energy density, induce important cosmological implications, in the picture of a modified Raychaudhury equation describing a non-singular universe, that allows us to explore the behaviour of our region of universe in early times. In this regard, by indicating $\left(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE}V}{\hbar c}\right)^2 = \sigma$, the modified Friedmann equations (47) and (48), together with the equation of state

$$\dot{H} = F\left(H\right) \,, \tag{52}$$

lead to obtain the following version of Raychaudhury equation:

$$\dot{H}-\frac{k}{a^2}=-\frac{3}{2}(1+\omega)\left[\frac{1}{2}\left(H^2+\frac{kc^2}{a^2}\right)-\frac{2}{3\sigma}\left[1-\left(1-\frac{\sigma}{2}\left(H^2+\frac{kc^2}{a^2}\right)\right)^{\frac{3}{2}}+C\right]\right] \times\frac{4}{\sigma}\,\frac{1-\left[1-\frac{\sigma}{2}\left(H^2+\frac{kc^2}{a^2}\right)\right]^{1/2}}{H^2+\frac{k}{a^2}}, \quad (53)$$

where C is given by (46) and $\omega=p/\rho$ is the equation of state. In the case $k=0$, taking account of (46), the Raychaudhury equation (53) becomes

$$\dot{H}=-\frac{3}{2}(1+\omega)\left[\frac{H^2}{2}-\frac{2}{3\sigma}\left[1-\left(1-\frac{\sigma H^2}{2}\right)^{\frac{3}{2}}+\frac{8\pi G}{3}\left(\Lambda+\frac{1}{4G\sigma\pi}\right)\right]\right] \times\left[\frac{4}{\sigma}\,\frac{1-\left(1-\frac{\sigma}{2}H^2\right)^{1/2}}{H^2}\right], \quad (54)$$

which leads to the following expression for the function (52):

$$F(H)=-\frac{6\pi}{\sigma H^2}(1+\omega)\left[\frac{H^2}{2}-\frac{2}{3\beta\sigma}\left[1-\left(1-\frac{\sigma H^2}{2}\right)^{\frac{3}{2}}+\frac{8\pi G}{3}\left(\Lambda+\frac{1}{4G\sigma\pi}\right)\right]\right] \times\left[1-\left(1-\frac{\sigma}{2}H^2\right)^{1/2}\right]. \quad (55)$$

Equations (53) and (54) predict the existence of a maximum value of the Hubble rate H, which is reached in a finite time and thus that there is no cosmological evolution beyond this point from a spacetime prospective. In agreement with the results obtained in [55, 56], the existence of maximum energy density and a general nonsingular evolution turns out to be independent of the equation of state and the spatial curvature k. The approach based on equations (52)–(55) that derive from the modified Friedmann equations, therefore, has the merit to avoid the presence of singularities in the evolution of the universe. Moreover – and this is a crucial result – it is compatible with the cyclic universe scenario, where the Big Bang and the Big Crunch are phenomena that occur in a cyclic manner with different initial and final densities each time [57].

In this picture, while the Hubble parameter $H(t)=\dot{a}/a$ oscillates periodically between positive and negative values from cycle to cycle, instead the Friedmann-Robertson-Walker scale factor $a(t)$ is characterized by a substantial increase from one cycle to the next, in the sense that $a(t)$ grows during the usual radiation, matter and dark energy dominated expanding phases, but shrinks very little during the contraction phases. Taking account the results obtained in [57], on the basis of the modified Friedmann equations (47) and (48), the evolutionary behaviour of the Friedmann-Robertson-Walker scale factor satisfies relation

$$a(t) \approx |t|^{1/\varepsilon} \ , \tag{56}$$

while the Hubble parameter $|H(t)|$ is proportional to $a^{-\varepsilon}$ during a phase with equation-of-state ε. In this picture, we can consider that the equation-of-state $\omega = p/\rho$ can be generalized in the form

$$\varepsilon_{\pm} = \frac{3}{2}\left(1 + \frac{p}{\rho}\right), \tag{57}$$

where the subscript – refers to the value of ε during the contracting phase (which is characterized by $H < 0$) and the subscript + indicates the value during the expanding phase (which is characterized by $H > 0$). By using the modified Friedmann equations (46) and (48), one obtains the following expressions for the pressure p and the energy density ρ appearing in the equation of state (57):

$$\begin{aligned} p = &- \Lambda - \frac{2}{8\pi G}\left(\dot{H} - \frac{kc^2}{a^2}\right)\frac{\sigma}{4}\frac{H^2 + \frac{kc^2}{a^2}}{1 - \left(1 - \frac{\sigma}{2}\left(H^2 + \frac{k}{a^2}\right)\right)^{1/2}} \\ &- \frac{3}{16\pi G}\left(H^2 + \frac{kc^2}{a^2}\right) - \frac{1}{4G\sigma\pi}\left[1 - \left(1 - \frac{\sigma}{2}\left(H^2 + \frac{kc^2}{a^2}\right)\right)^{3/2}\right], \end{aligned} \tag{58}$$

$$\rho = \frac{3}{16\pi G}\left(H^2 + \frac{kc^2}{a^2}\right) + \frac{1}{4G\sigma\pi}\left[1 - \left(1 - \frac{\sigma}{2}\left(H^2 + \frac{kc^2}{a^2}\right)\right)^{3/2}\right] + \Lambda\,. \tag{59}$$

In this way, the equation of state (57) becomes

$$\varepsilon_{\pm} = -\frac{3}{2}\left(\frac{\frac{2}{8\pi G}\left(\dot{H} - \frac{kc^2}{a^2}\right)\frac{\sigma}{4}\frac{H^2 + \frac{k}{a^2}}{1 - \left(1 - \frac{\sigma}{2}\left(H^2 + \frac{k}{a^2}\right)\right)^{1/2}}}{\frac{3}{16\pi G}\left(H^2 + \frac{kc^2}{a^2}\right) + \frac{1}{4G\pi\sigma}\left[1 - \left(1 - \frac{\sigma}{2}\left(H^2 + \frac{kc^2}{a^2}\right)\right)^{3/2}\right] + \Lambda}\right) \tag{60}$$

and the evolution law of the scale factor becomes

$$a(t) \approx |t|^{-\frac{2}{3}\left(\frac{\frac{3}{16\pi G}\left(H^2 + \frac{kc^2}{a^2}\right) + \frac{1}{4G\pi\sigma}\left[1 - \left(1 - \frac{\sigma}{2}\left(H^2 + \frac{kc^2}{a^2}\right)\right)^{3/2}\right] + \Lambda}{\frac{2}{8\pi G}\left(\dot{H} - \frac{kc^2}{a^2}\right)\frac{\sigma}{4}\frac{H^2 + \frac{k}{a^2}}{1 - \left(1 - \frac{\sigma}{2}\left(H^2 + \frac{k}{a^2}\right)\right)^{1/2}}}\right)} \tag{61}$$

Compatibly with the results obtained in [57], our model based on equations (60) and (61) determines the following predictions as regards the evolution of the universe. One deals with cyclic processes, where: during the expanding phases, at the beginning the Hubble parameter $H(t)$ is large and positive and then decreases; instead, when the expansion phase ends, one has the beginning of a contracting phase where $\varepsilon_- \gg 1$, in which one deals with a (non-singular) classical bounce phase, which is characterized by a rapid increase of the Hubble parameter $H(t)$ from a large negative value to a large positive value of approximately the same magnitude while $a(t)$ remains the same. In this situation, the universe enters the next expansion phase and a new cycle begins. As regards these cyclic phases of expansion and contraction, we must emphasize that here there is a non-singular connection between them, compatibly with the bouncing paradigms. The transition from contraction to expansion corresponds to a critical point where one deals with a minimal nonzero value of the scale factor a_0 and, by imposing the constraint $H\Big|_{a=a_0} = 0$ into the Friedmann equation (47), one obtains the following value for the critical energy density:

$$\rho_{\text{crit}} = \frac{1 + \dfrac{k}{2\pi a_0^2}\left[\dfrac{2}{3\sigma}\left[1 - \left(1 - \dfrac{\sigma}{2}\left(\dfrac{kc^2}{a_0^2}\right)\right)^{3/2}\right]\right]^{1/2}}{\dfrac{2}{3\sigma}\left[1 - \left(1 - \dfrac{\sigma}{2}\left(\dfrac{kc^2}{a_0^2}\right)\right)^{3/2}\right]} . \tag{62}$$

From equation (62), by imposing the constraint $1 - \left(1 - \frac{\sigma}{2}\left(\frac{kc^2}{a_0^2}\right)\right)^{3/2} \geq 0$, it follows that the minimal value for the scale factor is:

$$a_0 = \sqrt{\frac{\sigma kc^2}{2}} . \tag{63}$$

On the basis of equation (63), if the quantity σ tends to the square of Planck length, such state is not stable and the universe turns out to enter a bouncing behaviour at this regime.

As a consequence of the critical energy density (62), in the expansion phase of the universe, the scale factor obeys an exponential law of the form

$$a_0(t) \approx A_1 \exp\left(\frac{\mathcal{C}}{\sqrt{\left(\dfrac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2 + \left(\beta l_p^2 \dfrac{\Delta\rho_{qvE}V}{\hbar c}\right)^2}} t\right), \tag{64}$$

while, in the contraction phase, the scale factor satisfies equation

$$a_0(t) \approx A_2 \exp\left(-\frac{\mathcal{C}}{\sqrt{\left(\dfrac{\beta\hbar c}{\Delta\rho_{qvE}V}\right)^2 + \left(\beta l_p^2 \dfrac{\Delta\rho_{qvE}V}{\hbar c}\right)^2}} t\right), \tag{65}$$

where A_1, $\mathcal{C}$, A_2 are constants, A_1 and A_2 being proportional to the Planck length, and t is time. Equations (64) and (65), in the regime in which the generalized Compton wavelength tends to the Planck length and $\mathcal{C}$ tends to the Planck length, lead to results that turn out to be in agreement with the entropic force scenario equipped with a zero-point length correction to the gravitational potential provided in [54].

Moreover, during the expansion phase of each cycle, the evolution of the universe is characterized by different periods corresponding to the different values of ε_+, i.e. when $\varepsilon_+ = 2$ the dominant form of energy density is radiation and this situation occurs under the constraint

$$\left\{-\Lambda-\frac{2}{8\pi G}\left(\dot{H}-\frac{kc^2}{a^2}\right)\frac{\sigma}{4}\,\frac{H^2+\dfrac{kc^2}{a^2}}{1-\left(1-\frac{\sigma}{2}\left(H^2+\dfrac{k}{a^2}\right)\right)^{1/2}}-\frac{3}{16\pi G}\left(H^2+\frac{kc^2}{a^2}\right)\right.$$
$$\left.-\frac{1}{4G\sigma\pi}\left[1-\left(1-\frac{\sigma}{2}\left(H^2+\frac{kc^2}{a^2}\right)\right)^{3/2}\right]\right\}\Big/\left\{\frac{3}{16\pi G}\left(H^2+\frac{kc^2}{a^2}\right)\right.$$
$$\left.+\frac{1}{4G\sigma\pi}\left[1-\left(1-\frac{\sigma}{2}\left(H^2+\frac{kc^2}{a^2}\right)\right)^{3/2}\right]+\Lambda\right\}=\frac{1}{3}\,, \quad (66)$$

namely

$$-\frac{4}{3}\Lambda-\frac{2}{8\pi G}\left(\dot{H}-\frac{kc^2}{a^2}\right)\frac{\sigma}{4}\,\frac{H^2+\dfrac{kc^2}{a^2}}{1-\left(1-\dfrac{\sigma}{2}\left(H^2+\dfrac{k}{a^2}\right)\right)^{1/2}}$$
$$-\frac{1}{4\pi G}\left(H^2+\frac{kc^2}{a^2}\right)-\frac{1}{3G\sigma\pi}\left[1-\left(1-\frac{\sigma}{2}\left(H^2+\frac{kc^2}{a^2}\right)\right)^{3/2}\right]=0\,; \quad (67)$$

when $\varepsilon_+ = 3/2$ the dominant form of energy density is matter and this occurs under the constraint

$$-\Lambda-\frac{3}{16\pi G}\left(H^2+\frac{kc^2}{a^2}\right)-\frac{1}{4G\sigma\pi}\left[1-\left(1-\frac{\sigma}{2}\left(H^2+\frac{kc^2}{a^2}\right)\right)^{3/2}\right]$$
$$=\frac{2}{8\pi G}\left(\dot{H}-\frac{kc^2}{a^2}\right)\frac{\sigma}{4}\,\frac{\left(H^2+\dfrac{kc^2}{a^2}\right)}{\left[1-\left(1-\dfrac{\sigma}{2}\left(H^2+\dfrac{k}{a^2}\right)\right)^{1/2}\right]}\,; \quad (68)$$

finally, when $\varepsilon_+ \approx 0$, the dominant form of energy density is dark energy, under the constraint

$$\left\{-\Lambda-\frac{2}{8\pi G}\left(\dot{H}-\frac{kc^2}{a^2}\right)\frac{\sigma}{4}\,\frac{H^2+\dfrac{kc^2}{a^2}}{1-\left(1-\frac{\sigma}{2}\left(H^2+\dfrac{k}{a^2}\right)\right)^{1/2}}-\frac{3}{16\pi G}\left(H^2+\frac{kc^2}{a^2}\right)\right.$$
$$\left.-\frac{1}{4G\sigma\pi}\left[1-\left(1-\frac{\sigma}{2}\left(H^2+\frac{kc^2}{a^2}\right)\right)^{3/2}\right]\right\}\Big/\left\{\frac{3}{16\pi G}\left(H^2+\frac{kc^2}{a^2}\right)\right.$$
$$\left.+\frac{1}{4G\sigma\pi}\left[1-\left(1-\frac{\sigma}{2}\left(H^2+\frac{kc^2}{a^2}\right)\right)^{3/2}\right]+\Lambda\right\}=-1\,, \quad (69)$$

namely

$$-\frac{2}{8\pi G}\left(\dot{H}-\frac{kc^2}{a^2}\right)\frac{\sigma}{4}\frac{H^2+\dfrac{kc^2}{a^2}}{1-\left(1-\dfrac{\sigma}{2}\left(H^2+\dfrac{kc^2}{a^2}\right)\right)^{1/2}}=0\,. \tag{70}$$

In the light of equations (59)–(70), we can say that the different forms of evolution ruled by vacuum energy density, matter and dark energy density, are generated by a specific peculiar interplay of the discretization of spacetime associated with the energy fluctuations of the virtual sub-Planckian black holes of the variable quantum vacuum energy density and cosmological parameters. This means that it is the behaviour of the variable energy density characterizing the ultimate texture of the sub-Planckian black holes of the 3D DQV that determines what occurs in the different phases of evolution of the universe. And, therefore, in this picture, the perspective is opened that indeed dark energy density does not exist as a primary physical reality but is fixed by specific energy density fluctuations of of the lattice of the sub-Planckian black holes of the 3D DQV.

On the other hand, by following [58], during evolution, dark energy can be described through a scalar field potential of the 3D quantum vacuum $V(\Delta\rho_{qvE}^{DE})$. During the radiation- and matter-dominated phases, the quantum vacuum energy density fluctuations $\Delta\rho_{qvE}^{DE}$ generating the action of dark energy satisfy the constraint $V\left(\Delta\rho_{qvE}^{DE}\right)>0$ and here, as a consequence of a corresponding behaviour of $\Delta\rho_{qvE}^{DE}$, one deals with a transition from the accelerated expansion phase to the contraction phase when the scalar field potential of the 3D quantum vacuum $V(\Delta\rho_{qvE}^{DE})$ changes from $V\left(\Delta\rho_{qvE}^{DE}\right)>0$ to $V\left(\Delta\rho_{qvE}^{DE}\right)<0$. In this situation, the accelerated expansion phase is followed by a ultraslow contraction ruled by quantum vacuum energy density fluctuations that act as a sort of ekpyrotic field.

The scalar field potential $V\left(\Delta\rho_{qvE}^{DE}\right)$ of the fluctuations of the quantum vacuum energy density which generates the action of dark energy, has a crucial role in determining the specific processes occurring in each cycle, in the sense that it fixes the time when the domination of dark energy begins and thus the characteristic time scale for the duration of the expanding phase, it generates the contracting phase as well as it determines the total period of a cycle. Moreover, taking account of the results obtained in [57, 59], the quantum vacuum energy density fluctuations and the scalar field potential $V\left(\Delta\rho_{qvE}^{DE}\right)$, associated with the action of the dark energy, satisfy the following equation-of-state:

$$\varepsilon_{-}=3\,\frac{\frac{1}{2}\Delta\dot{\rho}_{qvE}^{DE}}{\frac{1}{2}\Delta\dot{\rho}_{qvE}^{DE}+V\left(\Delta\rho_{qvE}^{DE}\right)} \tag{71}$$

and the slow contraction characterizing each cycle can be associated to a scalar field potential depending on $\Delta\rho_{qvE}^{DE}$ and the granular features of the 3D DQV expressed by relation

$$V = -V_0 \exp\left[\frac{\Delta\rho_{qvE}^{DE}}{\Delta\rho_{qvE}^{3}V^2}\beta\frac{\hbar c^5}{G}\Big[\Big(\frac{\beta\hbar c}{\Delta\rho_{qvE}V}\Big)^2 + \Big(\beta l_p^2\frac{\Delta\rho_{qvE}V}{\hbar c}\Big)^2\Big]\right], \quad (72)$$

where V_0 is a constant. On the basis of equations (71) and (72), one can derive important consequences as regards the physical processes occurring during the contraction phase. In the light of the results obtained in [59], a peculiar solution is possible where the equation of state of the scalar field (57) converges to $\varepsilon_- = c^2/2\Delta\rho_{qvE}V$, determining a big value of ε event for short values of the quantum vacuum energy density fluctuations, thus resolving the homogeneity, isotropy, and flatness problems. We can therefore say that the discretization of the spatial geometry associated with the texture of sub-Planckian black holes of the variable energy density of the 3D DQV lead to a novel re-reading (and solution) of important issues of contemporary cosmology.

Moreover, we must underline that, in the cycles of evolution of the universe, the contraction phase turns out to be characterized by a shrinking of the scale factor $a(t)$ and an even bigger shrinking of the Hubble parameter $H(t)$ and, in this regard, it is the discretization of the spatial geometry associated with the texture of sub-Planckian black holes of the variable energy density of the 3D DQV that determines the features of these processes. In the light of equation (71) and (72), the homogeneous ekpyrotic field energy (which is associated with a value $\varepsilon_- \gg 1$) grows much faster, during the contracting phase, than all other components (matter, radiation, dark energy, gradient energy, spatial curvature, anisotropy). As a consequence of this peculiar behaviour, the universe is driven towards an ultra-uniform, ultra-flat state as the end of a cycle is approached and the beginning of a new cycle occurs. In these processes, the crucial fact is that the opportune fluctuations of the quantum vacuum energy density are the fundamental physical entities which determine the large value of ε_- and, in correspondence, the smoothing and flattening effect in space. The potential (72) ultimately depending on the discretization of the spatial geometry of the texture of sub-Planckian black holes of the variable energy density has, therefore, essential roles in each state of each cycle: it provides the source of the dark energy that rules the accelerated expansion of space in the expansion phase like the current epoch; it generates the transition from accelerated expansion to slow contraction; it is the intermediary entity that leads the universe from the contraction phase to the next accelerated phase of the new cycle; and, finally, it can be seen as the origin of the hot matter-radiation domination in the phase before the energy density fluctuations of the 3D DQV acting as dark energy take over.

In our approach, the net result determined by the fluctuations of the vacuum associated with the texture of the sub-Planckian black holes of the variable quantum vacuum energy density is that the Friedmann-Robertson-Walker scale factor $a(t)$ grows from cycle to cycle and thus that the universe can be seen as a timeless phenomenon in dynamic equilibrium where each cycle of evolution leads to the creation of new systems of space and matter which have no end. This is the fundamental perspective opened by our model based on the modified Friedmann equations (47) and (48).

In our model, as a consequence of the cyclic processes above described, also the Hubble parameter $H(t)$ has a cyclic behaviour. Finally, we emphasize that our model turns out to be in agreement with the current values of astronomical parameters. In fact, if the minimum value of the Hubble radius r_H is taken as $\sim 10^{-25}$ cm, corresponding to a maximum value of the Hubble parameter of $|H| \sim 10^{10}$ GeV and we assume that the temperature is $T \sim 10^{15}$ GeV at the start of the process of creation of new matter in the processes of expansion, one finds that the scale factor $a(t)$ increases by a factor $e^{120/\varepsilon_+} \sim e^{60}$ by the time the temperature reaches today's cosmic microwave background temperature and, in correspondence, one deals with an increase of the Hubble radius by a factor of $a^{\varepsilon_+} \sim e^{120}$ from a microscopic size of 10^{-25} cm to the current value of the Hubble radius 10^{28} cm.

In summary, our model predicts that each cycle ends with a non-singular bounce, where the value of the Hubble parameter $H(t)$ and the density of matter and radiation have returned to the values they had a cycle before. This implies the beginning of a new period of oscillation in $H(t)$ with the creation of new fresh matter and radiation, that is coherent with results obtained in the context of inflationary approaches.

5 Conclusions

If in the 1960s Mead underlined the necessity of exploring gravitational interaction at very short distances [60] and ten years later Hawking realized the crucial role of "trans-Planckian" regime of infinitely small distances for the understanding of gravity in the thermodynamics of black holes [61], recent research show how the generalized uncertainty principle represents a fundamental heuristic tool in order to investigate quantum gravity effects. The existence of a minimal length scale introduced by generalized uncertainty relations lead to obtain modified forms of the Friedmann equations, showing how quantum effects induced at high energies can affect the dynamics of a FLRW universe at early times. In this regard, in particular, by starting from a quadratic generalized uncertainty relation of the form

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1 + \alpha_Q^2 \frac{l_p^2}{\hbar^2}\right), \tag{73}$$

recently a model of modified Friedmann equations has been developed, that lead to predictions that are in agreement with the standard ΛCDM cosmological model in a picture where the contribution of Planck scale does not act as an alternative dark energy fluid and $\alpha_Q^2 < 10^{59}$ which is one of the few constraints obtained with cosmological data [62]. In the light of the impact of generalized uncertainty principle in quantum gravity and cosmology and in the light of the problems of the standard ΛCDM cosmological model, it becomes natural to consider, in the picture of alternative generalized uncertainty relations, scenarios of modified Friedmann equations, that are compatible with alternative cosmological models.

In this paper, a novel perspective of modified Friedmann equations has been explored in the picture of a 3D DQV defined by a variable energy density. By starting from the alternative form of the generalized uncertainty relation (1), we have considered a model based on sub-Planckian black holes of the variable energy density of the 3D DQV characterized by the generalized Compton wavelength (4) as fundamental scale of nature, that leads to an unification of microphysics of elementary particles and macrophysics of black holes. The texture of these sub-Planckian black holes of the variable energy density of the 3D DQV implies a discretization of spacetime associated with the fundamental scale represented by the generalized Compton wavelength and, above all, in the cosmological regime, leads to modified Friedmann equations where the departure degree of Friedmann equations from the classical behavior predicted by general relativity is linked with an opportune term depending of the generalized Compton wavelength and parameters characterizing the sub-Planckian black holes. These modified Friedmann equations in the background of the sub-Planckian black holes allow us to go beyond the results of standard cosmological model, by implying the absence of singularities in the evolution of the universe and predicting a cyclic universe scenario, where the Big Bang and the Big Crunch are phenomena that occur in a cyclic manner, in a picture where the different forms of evolution ruled by energy density of the vacuum, matter and dark energy density, are generated by a specific peculiar interplay of the discretization of spacetime associated with the energy fluctuations of the virtual sub-Planckian black holes of the variable quantum vacuum energy density and cosmological parameters. From the modified Friedmann equations ruling the background of the sub-Planckian black holes of the variable energy density of the 3D DQV, a cyclic model of the universe emerges, that leads to novel perspectives of rereading of important issues of cosmology, providing results that are compatible with current research.

Declarations

The author states that, as regards this paper, there is no conflict of interests/competing interests.

References

[1] B.J. Carr (2016) The Black Hole Uncertainty Principle Correspondence. *Springer Proceedings Physics* **170** 159-167.

[2] M.J. Lake, B. Carr (2016) The Compton-Schwarzschild relations in higher dimensions. arXiv:1611.01913 [gr-qc].

[3] M.J. Lake, B. Carr (2018) Does Compton/Schwarzschild duality in higher dimensions exclude TeV quantum gravity? *Int. J. Mod. Phys. D* 1930001.

[4] M.J. Lake, B. Carr (2015) The Compton-Schwarzschild correspondence from extended de Broglie relations. *JHEP* **1511** 105.

[5] B.J. Carr, J.R. Mureika, P. Nicolini (2015) Sub-Planckian black holes and the generalized uncertainty principle. *JHEP* **2015** 07052.

[6] J. Carr, L. Modesto, I. Prémont-Schwarz (2011) Generalized uncertainty principle and self-dual black holes. arXiv:1107.0708 [gr-qc].

[7] M. Maggiore (1993) The algebraic structure of the generalized uncertainty principle. *Phys. Lett. B* **319** 83-86.

[8] A. Kempf, G. Mangano, R.B. Mann (1995) Hilbert space representation of the minimal length uncertainty relation. *Phys. Rev. D* **52**(2) 1108-1118.

[9] F. Scardigli (1999) Generalized uncertainty principle in quantum gravity from micro-black hole gedanken experiment. *Phys. Lett. B* **452** 39-44.

[10] R.J. Adler, D.I. Santiago (1999) On gravity and the uncertainty principle. *Mod. Phys. Lett. A* **14** 1371-1381.

[11] S. Capozziello, G. Lambiase, G. Scarpetta (2000) Generalized uncertainty principle from quantum geometry. *Int. J. Theor. Phys.* **39** 15-22.

[12] S. Hossenfelder (2013) Minimal length scale scenarios for quantum gravity. *Living Rev. Relativ.* **16** 2.

[13] R. Casadio, F. Scardigli (2014) Horizon wave function for single localized particles: GUP and quantum black-hole decay. *Eur. Phys. J. C* **74** 2685.

[14] L. Petruzziello, F. Illuminati (2021) Quantum gravitational decoherence from fluctuating minimal length and deformation parameter at the Planck scale. *Nat. Commun.* **12** 4449.

[15] A.F. Ali, S. Das, E.C. Vagenas (2009) Discretization of space from the generalized uncertainty principle. *Phys. Lett. B* **678** 497-499.

[16] A. Alonso-Serrano, M.P. Dabrowski, H. Gohar (2018) Generalized uncertainty principle impact onto the black holes information flux and the sparsity of Hawking radiation. *Phys. Rev. D* **97** 044029.

[17] F. Scardigli, G. Lambiase, E. Vagenas (2017) GUP parameter from quantum corrections to the Newtonian potential. *Phys. Lett. B* **767** 242-246.

[18] R. Casadio, F. Scardigli (2014) Horizon wave-function for single localized particles: GUP and quantum black hole decay. *Eur. Phys. J. C* **74** 2685.

[19] P. Chen, Y.C. Ong, D.-H. Yeom (2015) Black Hole Remnants and the Information Loss Paradox. *Phys. Rep.* **603** 1-45.

[20] F. Scardigli, C. Gruber, P. Chen (2011) Black hole remnants in the early universe. *Phys. Rev. D* **83** 063507.

[21] M. Blasone, G. Lambiase, G.G. Luciano, L. Petruzziello, F. Scardigli (2019) Heuristic derivation of the Casimir effect from generalized uncertainty principle. arXiv:1902.02414.

[22] D. Fiscaletti (2015) *The timeless approach. Frontier perspectives in 21st century physics*, World Scientific, Singapore.

[23] D. Fiscaletti (2016) About dark energy and dark matter in a three-dimensional quantum vacuum model. *Found. Phys.* **46**(10) 1307-1340.

[24] D. Fiscaletti, A. Sorli (2016) About a three-dimensional quantum vacuum as the ultimate origin of gravity, electromagnetic field, dark energy ... and quantum behaviour. *Ukr. J. Phys.* **61**(5) 413-431.

[25] D. Fiscaletti, A. Sorli (2016) Dynamic quantum vacuum and relativity. *Annales UMCS Sectio AAA: Physica* LXXI, pp. 11-52.

[26] D. Fiscaletti, A. Sorli (2016) About electroweak symmetry breaking, electroweak vacuum and dark matter in a new suggested proposal of completion of the Standard Model in terms of energy fluctuations of a timeless three-dimensional quantum vacuum. *Quantum Phys. Lett.* **5**(3) 55-69.

[27] D. Fiscaletti, A. Sorli (2017) Quantum vacuum energy density and unifying perspectives between gravity and quantum behaviour of matter. *Annales de la Fondation Louis de Broglie* **42**(2) 251-297.

[28] D. Fiscaletti, A. Sorli (2018) Quantum relativity: variable energy density of quantum vacuum as the origin of mass, gravity and the quantum behaviour. *Ukr. J. Phys.* **63**(7) 623-644.

[29] D. Fiscaletti (2020) About dark matter as an emerging entity from elementary energy density fluctuations of a three-dimensional quantum vacuum. *J. Theor. Appl. Phys.* **14** 203-222.

[30] D. Fiscaletti, A. Sorli (2023) Generalized uncertainty relations, particles, black holes, and Casimir effect in the three-dimensional quantum vacuum. *Theor. Math. Phys.* **214**(1) 132-151.

[31] C. Rovelli (2003) Loop quantum gravity. *Phys. World* **7**(11) 1-5.

[32] C. Rovelli (2008) Loop quantum gravity. *Living Rev. Relativ.* **11** Article No. 5.

[33] C. Rovelli (2011) A new look at loop quantum gravity. *Class. Quantum Grav.* **28**(11) 114005.

[34] C. Rovelli (2004) *Quantum Gravity*, Cambridge University Press, Cambridge.

[35] Y. Jack Ng (2007) Holographic Foam, Dark Energy and Infinite Statistics. *Phys. Lett. B* **657** 10-14.

[36] Y. Jack Ng (2008) Spacetime foam: from entropy and holography to infinite statistics and non-locality. *Entropy* **10** 441-461.

[37] Y. Jack Ng (2010) Holographic quantum foam. arXiv:1001.0411v1 [gr-qc].

[38] Y. Jack Ng (2011) Various facets of spacetime foam. arXiv:1102.4109.v1 [gr-qc].

[39] N. Margolus, L.B. Levitin (1998) The Maximum Speed of Dynamical Evolution. *Physica D* **120** 188-195.

[40] L. Susskind (1995) The World as a Hologram. *J. Math. Phys.* **36** 6377-6396.

[41] J.D. Bekenstein (1973) Black Holes and Entropy. *Phys. Rev. D* **7** 2333-2346.

[42] S. Hawking (1975) Particle Creation by Black Holes. *Commun. Math. Phys.* **43** 199-220.

[43] S.B. Giddings (1992) Black Holes and Massive Remnants. *Phys. Rev. D* **46** 1347-1352.

[44] R. Bousso (2002) The Holographic Principle. *Rev. Mod. Phys.* **74** 825-874.

[45] A.J.M. Medved, E.C. Vagenas (2004) When conceptual worlds collide: The GUP and the BH entropy. *Phys. Rev. D* **70** 124021.

[46] R.J. Adler, P. Chen, D.I. Santiago (2001) The Generalized uncertainty principle and black hole remnants. *Gen. Relativ. Gravit.* **33** 2101.

[47] M. Cavaglia, S. Das, R. Maartens (2003) Will we observe black holes at LHC? *Class. Quantum Grav.* **20** L205.

[48] M. Cavaglia, S. Das (2004) How classical are TeV scale black holes? *Class. Quantum Grav.* **21** 4511.

[49] C. Adami (2004) The Physics of Information. arXiv:quant-ph/0405005.

[50] E.P. Verlinde (2011) On the origin of gravity and the laws of Newton. *JHEP* **1104** 029.

[51] T. Padmanabhan (2010) Equipartition of energy in the horizon degrees of freedom and the emergence of gravity. *Mod. Phys. Lett. A* **25**(14) 1129-1136.

[52] P. Nicolini, E. Spallucci, M.F. Wondrak (2019) Quantum Corrected Black Holes from String T-Duality. *Phys. Lett. B* **797** 134888.

[53] P. Gaete, K. Jusufi, P. Nicolini (2022) Charged black holes from T-duality. *Phys. Lett. B* **835** 137546.

[54] K. Jusufi, A. Sheykhi (2023) Entropic corrections to Friedmann equations and bouncing universe due to the zero-point length. *Phys. Lett. B* **836** 137621.

[55] A. Awad, A.F. Ali (2014) Minimal length, Friedmann equations and maximum density. *JHEP* **06** 093.

[56] M. Salah, F. Hammad, M. Faizal, A. Farag Ali (2017) Non-singular and cyclic universe from the modified GUP. arXiv:1608.00560v3 [gr-qc].

[57] A. Ijjas, P.J. Steinhardt (2019) A new kind of cyclic universe. *Phys. Lett. B* **795** 666-672.

[58] A Feoli, E. Benedetto, M. Capriolo, F. Feleppa (2017) Is dark energy due to an excited quantum state of the universe? *Eur. Phys. J. Plus* **132** 211.

[59] A. Ijjas, P.J. Steinhardt (2022) Entropy, black holes, and the new cyclic universe. *Phys. Lett. B* **824** 136823.

[60] C. Mead (1964) Possible Connection Between Gravitation and Fundamental Length. *Phys. Rev.* **135** B849-B862.

[61] S. Hawking (1975) Particle creations by black holes. *Commun. Math. Phys.* **43** 199-220; [Erratum: *Commun. Math. Phys.* **46** 206 (1976)].

[62] S. Giardino, V. Salzano (2021) Cosmological constraints on GUP from modified Friedmann equations. *Eur. Phys. J. C* **81** 110.