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Cosmic-Ray Transport Equation: Solutions and Semi-Empirical Models

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Abstract. Galactic radiation is the most penetrating type of cosmic radiation in the Earth's atmosphere. The galactic cosmic rays (GCRs), passing through the heliosphere to reach to the upper atmosphere, change their energy, direction of propagation and may even be absorbed along the way. The galactic cosmic-ray transport in the heliosphere is described by the well-known transport equation developed by American solar astrophysicist Eugene Newman Parker. Parker has shown that – in the framework of statistical physics – the random walk of cosmic-ray particles is a Markoff process, describable by a Fokker–Planck equation (FPE). Gleeson and Axford (1967) subsequently rederived the Parker transport equation for cosmic rays starting with a Boltzmann type equation for a radial geometry.

In this work are discussed the most widely used analytical solutions to the Parker transport equation. It is also shown that an approximate solution of Fisk and Axford (1969) to the transport equation is the basis of widely used semi-empirical models. The further development of this type of models is related to finding functional dependencies between the parameters in the models and different heliospheric and solar variables. These studies are important for deriving more accurate semi-empirical models describing and predicting GCR spectra during the solar cycle.

KEY WORDS: galactic cosmic radiation, cosmic-ray transport equation, solar cycle modulation, semi-empirical models.

1 Introduction

Cosmic radiation is a form of ionizing radiation that originates in outer space. It consists of high-energy charged particles, x-rays and gamma rays. Although the higher ultraviolet part of the electromagnetic spectrum is ionizing radiation ultraviolet photons coming from the Sun is not considered cosmic radiation [1]. High-energy particles with intrinsic mass are known as "cosmic rays" while photons, which are quanta of electromagnetic radiation (and so have no intrinsic mass) are known by their common names, such as gamma rays or X-rays, depending on their photon energy [2].

The term "ray" is due to a historical accident, as at first cosmic rays were wrongly thought to be mostly electromagnetic radiation [2]. Thus, since the discovery of the cosmic radiation the general picture has changed several times: The primary radiation was at first believed to be a gamma radiation, later a beta radiation, until it was proved to consist mainly of protons [3].

Before cosmic rays (CRs) interact in the atmosphere they are known as primary cosmic rays (PCRs). In the vicinity of the Earth the majority of high-energy particles are protons (hydrogen nuclei), about 10% are helium nuclei (nuclear physicists usually call them alpha particles) and 1% are neutrons or nuclei of heavier elements (these percentages vary over the energy range of cosmic rays). Together, these constitute 99% of the cosmic radiation, and electrons and photons make up the remaining 1% [4, 5].

Cosmic rays include the galactic cosmic rays and other classes of energetic particles, including solar energetic particles and particles accelerated in interplanetary space [6]. However, the term "cosmic rays" is often used to refer only to galactic cosmic rays, which originate in sources outside the solar system [2, 6].

The energy spectrum of galactic cosmic nuclei for energy E above 10^{10} eV/nucl. follows a power law $D(E) = AE^{-\gamma}$, $D(E)$ is the differential flux for given energy E and γ – the spectral index. At energies less than 10 GeV the spectra of protons, nuclei and electrons are influenced by interplanetary and terrestrial magnetic fields [7]. Unlike cosmic rays, gamma rays are not deflected by magnetic fields because they have no charge.

It is believed that below the knee cosmic rays are of galactic origin and their propagation is limited in magnetic fields of our galaxy. As CR motion is randomized by irregular interstellar fields, galactic cosmic rays appear to be highly isotropic in the solar neighborhood [7, 8].

2 Cosmic-Ray Propagation in the Heliosphere: Parker Transport Equation and Gleeson-Axford Model Equations

The galactic cosmic-ray (GCR) transport in the heliosphere is described by the well-known transport equation developed for the first time by astrophysicist Eugene Parker. This equation is a simplified version of Fokker-Planck transport equation, $i.e.$, it is a Fokker-Plank type. Parker [9–11] develops the first hydrodynamic model of the coronal expansion of the sun plasma and introduces the term solar wind [12]. Using the model, he describes and explains the galactic cosmic-ray behavior in the interplanetary region. When entering the heliosphere, the behavior of galactic cosmic rays is affected by the solar wind and the interplanetary magnetic field. This influence which is seen, e.g. in the change of cosmic ray intensity and spectrum is called the *solar modulation* [12].

The interplanetary magnetic field has two components: regular part, which has a spiral structure, moves outward from the sun and is as frozen in to solar wind, and irregular part. The cosmic-ray diffusion is caused by the scattering of the cosmic-ray particles by the irregularities in the magnetic field [13]. Diffusion-like propagation together with drift, convection and energy change are the basic modulation mechanisms responsible for the 11-year and 22-year solar cycle variations in the intensity of galactic cosmic rays. These four major mechanisms were incorporated by Parker into the cosmic-ray transport equation [14]. For a radial wind of constant speed V and diffusion with an isotropic diffusion coefficient Parker's transport equation [14] is:

$$\frac{\partial U}{\partial t} + \frac{\mathrm{V}}{r^2}\frac{\partial}{\partial r}\left(r^2 U\right) - \frac{2\mathrm{V}}{3r}\frac{\partial}{\partial T}\left(\alpha T U\right) - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k \frac{\partial U}{\partial r}\right) = 0\,, \qquad (1)$$

where $U \equiv U(r, T, t)$ is the differential number density (number of particles per unit volume having kinetic energy T) at time t and r is the heliocentric radius. The solar wind speed is given by V, the particle diffusion coefficient by k, and $\alpha = (T + 2E_0)/(T + E_0)$ with E_0 – particle rest energy (α – troublesome factor [15]). For nonrelativistic particles $\alpha(T) = 2$ and $\alpha(T) = 1$ for extreme relativistic particles [16] so that $1 \leq \alpha \leq 2$ [17].

Changes in the heliosphere over the solar cycle occur slowly with respect to the transit time of cosmic rays and of the solar wind so that a quasi-steady state can be assumed [18]. The time-derivative can be neglected in Eq. (1) and cosmic-ray transport equation for stationary modulation in a spherically symmetric solar wind with constant speed V, under the assumption that the diffusion is isotropic [19, 20] has a form:

$$\frac{V}{r^2}\frac{\partial}{\partial r}\left(r^2 U_T\right) - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k \frac{\partial U_T}{\partial r}\right) - \frac{2V}{3r}\frac{\partial}{\partial T}\left(\alpha T U_T\right) = 0\,. \qquad (2)$$

Here, $U_T = U(r, T)$, $[U_T] = \text{particles}/(\mathrm{m}^3\, T)$ [16]. The terms in Eq. (2) describe, from left to right, the convection, diffusion of the particles and energy loss in the expanding solar wind.

In the derivation of the cosmic-ray transport equation Parker has demonstrated that – in the framework of statistical physics – the random walk of cosmic-ray particles is a Markoff process, describable by a Fokker–Planck equation (FPE) [21]. In its original formulation, Parker's transport-equation is calculated for the particle density for unit space and energy $U(\boldsymbol{x}, T, t)$ [14, 22, 23].

An alternative theory of cosmic-ray propagation developed by Axford [24] considers the Boltzmann equation of cosmic-ray gas in the interplanetary magnetic field [25]. Gleeson and Axford [26] subsequently rederived the transport equation for cosmic rays (Eq. 1) starting with a Boltzmann type equation for a radial geometry. In a completely different from Parker [14] detailed analysis, Gleeson and Axford [26] postulate the presence of small-scale irregularities superposed on the average interplanetary magnetic field B throughout the solar modulation region and study the motion of cosmic rays under scattering by these irregularities which move radially outward with the solar wind [25].

In the steady state, CR transport equation derived by Gleeson and Axford [26] has a form:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 S_T\right) = -\frac{1}{3}V\frac{\partial^2}{\partial r \partial T}\left(\alpha T U_T\right), \tag{3}$$

where $S_T = S(r,T)$ is the radial differential current density or streaming

$$S_T = VU_T - k\frac{\partial U_T}{\partial r} - \frac{V}{3}\frac{\partial}{\partial T}\left(\alpha T U_T\right). \tag{4}$$

Eliminating S_T between Eq. (3) and Eq. (4), the Parker transport Eq. (2) is obtained for the differential number density U_T [26]. Thus Gleeson and Axford [26] integrating the Boltzmann equation in the wind frame obtain two equations, Eq. (3) and Eq. (4), which correctly embodied the effects of adiabatic deceleration but are restricted to isotropic diffusion in a spherically symmetric medium [22]. In deriving Eq. (1) Parker [14] used the concept of adiabatic deceleration, whereas in deriving Eqs. (3), (4) and (1), Gleeson and Axford [26] used the concept of radially moving magnetic "scatterers" [27].

Some authors describe the modulation of GCRs re-expressing Eqs. (2)–(4) in terms of momentum, p [16, 28–30]:

$$\frac{V}{r^2}\frac{\partial}{\partial r}\left(r^2 U_p\right) - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k\frac{\partial U_p}{\partial r}\right) - \frac{2V}{3r}\frac{\partial}{\partial p}\left(pU_p\right) = 0, \tag{5}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 S_p\right) = -\frac{1}{3}V\frac{\partial^2}{\partial r \partial p}\left(pU_p\right), \tag{6}$$

$$S_p = VU_p - k\frac{\partial U_p}{\partial r} - \frac{V}{3}\frac{\partial}{\partial p}\left(pU_p\right). \tag{7}$$

The relation between the number density per unit of momentum, U_p, and the number density per unit of kinetic energy, U_T, is given by [31, 32]

$$U_p \mathrm{d}p = U_T \mathrm{d}T. \tag{8}$$

The relation between the particle kinetic energy T, the momentum p and the total energy E is [33]

$$E^2 = (pc)^2 + \left(m_0 c^2\right)^2, \tag{9}$$

where m_0 is the rest-mass of the particle and c is the speed of light. The quantity $E_0 = m_0 c^2$ is called the rest-mass energy. The total energy, E, is the sum of kinetic energy, T, and rest energy, E_0, *i.e.*, $E = T + E_0$, where rest-mass energy $E_0 = 938$ MeV (≈ 1 GeV) for protons, and 511 keV for electrons [32].

From Eq. (8), using Eq. (9) and taking into account that $\beta = v/c = \sqrt{T(T+2E_0)}/(T+E_0)$, we obtain [15, 32, 34]:

$$\frac{U_p}{U_T} = \beta c\,. \tag{10}$$

In practice we measure number of particles w.r.t. kinetic energy intervals and not momentum intervals [34].

3 Solutions of the Cosmic-Ray Transport Equation and Semi-Empirical Models

Parker's transport-equation in its original formulation for the particle density $U(\boldsymbol{x}, T, t)$ is given in very general form [14]:

$$\begin{aligned} &\frac{\partial U}{\partial t} + \frac{\partial}{\partial x_i}(U v_i) + \frac{\partial}{\partial T}\left[U \frac{\partial T}{\partial t}\right] - \frac{\partial}{\partial x_i}\left(k_{ij}\frac{\partial U}{\partial x_j}\right) = 0\,, \\ &\frac{1}{T}\frac{\partial T}{\partial t} = -\frac{\alpha(T)}{3}\frac{\partial v_i}{\partial x_i}\,. \end{aligned} \tag{11}$$

The first equation in (11) is five-dimensional, because of particle density $U(\mathbf{x}, T, t)$ depends on three spatial coordinates, plus kinetic energy T and time t.

3.1 Solutions of the cosmic-ray transport equation

The transport equation is a second-order partial differential equation of parabolic type [32] and is difficult to solve analytically. Its solution is found numerically. Methods of obtaining numerical solutions began with Fisk which solved the spherically-symmetric Fokker-Planck equation

$$\begin{aligned} \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 V U_T\right) - \frac{1}{3}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 V\right)\right]\left[\frac{\partial}{\partial T}\left(\alpha T U_T\right)\right] \\ = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k \frac{\partial U_T}{\partial r}\right), \end{aligned} \tag{12}$$

numerically, using the Crank-Nicholson technique [20]. Assuming that are known: the diffusion coefficient $k \equiv k(r, T)$, solar wind speed $V \equiv V(r)$, and the unmodulated spectrum $U_0(T, R)$, Fisk [20] developed a one-dimensional (1D) spherically symmetric steady-state model equation with radial distance r as the only spatial variable ($r = R$ is outer boundary of the modulating region, $i.e.$, some radial distance where the modulation becomes negligibly small). Later on many numerical models of various complexity were developed by different authors. A brief history of the numerical models, which compute cosmic-ray intensities in the heliosphere, is given in Manuel's PhD Thesis [35]. The cosmic-ray transport equation can only be solved numerically, it does not have an analytical solution in its most complex form. However, fifty years ago, numerical solutions of the transport equation were difficult to do and time-consuming on slow computers, so useful approximate analytical solutions of the full transport equation were obtained that can be applied directly to the observations with satisfactory results [32,36]. Although no analytic solution can be applied to the observational data through the whole energy range of interest [37].

There are many approximate solutions of the full cosmic-ray transport equation available. Moraal [32] summarizes the hierarchy of the approximations in increasing order of complexity. The simplest analytical approximations are Convection-Diffusion: $VU - \kappa\partial U/\partial r = 0$ and Force-Field: $CVU - \kappa\partial U/\partial r = 0$, C is the Compton-Getting factor, V – solar wind speed and k – an isotropic diffusion coefficient. Convection-diffusion approximation, the lowest-order approximation, was the basic equation used in earliest theories that neglected the effects of particle energy change [20, 38, 39]. It basically says that the diffusive flux is equal to the convective flux, $i.e.$, $VU = \kappa\partial U/\partial r$.

By doing many approximations (the solar wind moves radially with a constant speed, the diffusion coefficient is isotropic, the density distribution is spherically symmetric, there is no drift and the system is quasi-stationary, $i.e.$, $\partial U/\partial t = 0$) and based on the experimental result, Gleeson and Axford [36] find that for energy $T \gtrsim 200$ MeV/nucl. the radial streaming S is very small and, therefore, the term S_T can be neglected in Eq. (4). In this case, when $S_T = 0$, the force field equation from Eq. (4) is obtained

$$CVU = k\partial U/\partial r\,, \tag{13}$$

$C = [1 - (1/3U)(\partial/\partial T)(\alpha TU)]$ is the Compton-Getting coefficient. The force field approximation shows that the cosmic-ray flux needs the Compton-Getting correction. Actually, Eq. (13) expresses the balancing of the internal diffusion flow with the corrected external convective flow [32].

Later Gleeson and Urch [40] gave a much simpler and more transparent re-derivation of the force field solution that produces the so-called "modulation potential" ϕ [32, 41, 42]

$$\phi(r) \equiv \int_r^R \frac{V(r')}{3k_1(r')} dr'\,. \tag{14}$$

In this quasi-analytical solution the whole diffusion-convection process can be described using a single parameter – the modulation potential ϕ [23].

The force field approximation had been the most widely used and was surpassed only when numerical models became available [43]. However, because of its simplicity the force field approach is still and today widely used model for studying the effects of solar modulation, although several works underline its incapability correct reproduce the physical process in the heliosphere (see discussion in Refs. [23,41]). In Ref. [32] Moraal summarizes the hierarchy of approximation solutions to the Parker transport equation in increasing order of complexity. The two simplest solutions to the Parker transport equation are the convection-diffusion and the force field solution [32]. Moraal [32] indicates as the third level of approximation the numerical solution of the steady-state, spherically symmetric (or one-dimensional) transport equation, $i.e.$, the equation that includes adiabatic energy losses in its energy-dependent term. In addition, Moraal [32] notes that the Compton-Getting effect, $i.e.$, the proper transformation from the solar wind to the stationary frame, is also implicitly included in this model equation.

The one-dimensional solution to the Parker transport equation stands on some of the same approximations as the force field equation, except for the fact that the force field solution replaces adiabatic energy losses by a simulated energy loss, while the steady-state, spherically symmetric (or one-dimensional) transport equation includes adiabatic energy losses correctly [41,42].

Cabbalero-Lopez [41] and Moraal [32] summarize that at 1 AU the force field solution is a much better approximation to the full solution than the convection-diffusion solution, because the former includes energy loss effects. With increasing radial distance, the force field approximation gets progressively worse, but the convection-diffusion solution improves [32, 41]. The reason is that the force field solution progressively over-estimates the true, adiabatic energy loss [32,41]. For the same reason, the convection-diffusion approximation improves because energy losses become progressively smaller with increasing radial distance [32,41]. In Refs. [16,32,41] can be seen that the force field solution is a very good approximation to the 1D cosmic-ray solution for energies above about 0.1 GeV at $r = 1$ AU. But the force field approximation fails to estimate the correct adiabatic losses at low energies [16].

The Convection-Diffusion, Force-Field and the 1D approximate solution of the transport equation assume a spherically symmetric heliosphere (there is no dependence of the flux on the θ and φ coordinates), ignoring effects from higher-order, such as the geometry of the heliospheric magnetic field (HMF), drift motions in this field and in its wavy current sheet, and acceleration in traveling shocks and the termination shock of the heliosphere, which can only be studied with numerical solutions of the transport equation [16, 32]. For this reason, in these models, it only makes sense to study variations on a monthly basis (approximately over one Solar rotation ~ 27 days) [16]. In addition, the equations of the elementary theory of GCR modulation (see Eqs. (2)–(4)) include parametric dependencies and values of the parameters that we still don't know very well: the interstellar cosmic-ray spectrum in low-energy region, the diffusion coefficient and its dependence on radial distance and energy, radial dependence of the solar wind speed. In principle, we determine the particle number density U and radial streaming S when these parameters (the solar wind speed, the diffusion coefficient, and the interstellar cosmic-ray spectrum) are specified [14, 20, 26, 36, 39, 44, 45]. The mentioned parameters are not sufficiently well known throughout the heliosphere but their combination in the approximate models lead to useful analytical solutions, which can be applied directly to the observations.

In the article is shown that a few analytic solutions are available with simple form of the diffusion coefficient k, with constant V (the solar wind speed) and with constant α [39, 44, 46]. Gleeson and Axford [36] point out that these solutions are helpful as illustrations of certain aspects of the involved phenomena but are not immediately useful in the interpretation of observations. For example, the typical values of ϕ for solar minimum periods are below 400 MV, while periods of solar maximum have ϕ values above 800 MV [47]. Kelly Lave [47] notes that the energy spectra of different GCR species are sometimes best fit using slightly different values of ϕ and said that this is unsurprising since the spherically-symmetric Fokker-Planck Eq. (12) over-simplifies the solar environment by describing it as spherically-symmetric with no differences in the drift directions of particles in the $A > 0$ and $A < 0$ phases [47]. The Force-Field same as the 1D cosmic-ray solution does not take into account the complex structure of the magnetic field [42].

3.2 Semi-empirical models

In the past 50 years many analytical and semi-analytical solutions have been found. Because of its simplicity as well as due to good correspondence with experimental data, very often used are spherically symmetric study-state[1] solutions such as the "*Force-Field*" [36, 39]. Fisk and Axford [39] show that when $S_T \approx 0$, Eq. (4) can be solved directly for number density U_T assuming that V and α are constants, $k = k_0 T^a r^b$, $(b > 1)$, and that $U \to AT^{-\gamma}$ as $r \to \infty$. This solution is in a form [39]:

$$U_T = AT^{-\gamma}\left(1 - \frac{a\alpha}{3\,(1-b)}\frac{Vr}{k_0 T^a r^b}\right)^{\frac{1+\alpha(\gamma-1)/3}{-a\,\alpha/3}} . \tag{15}$$

The differential density U can be defined as a function of rigidity, P, as well U_P, with the appropriate transformation $U_P = (Ze/c)U_p$ [32]; note that the formal definition of rigidity is $P = pc/(Ze)$ [32]. If re-expressing Eq. (5) in terms of rigidity, P, the rigidity forms of Eqs. (6)–(7) are:

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 S_P) = -\frac{1}{3}V\frac{\partial^2}{\partial r\partial P}(PU_P)\,, \tag{16}$$

$$S_P = VU_P - k\frac{\partial U_P}{\partial r} - \frac{V}{3}\frac{\partial}{\partial P}(PU_P)\,. \tag{17}$$

[1] A steady-state solution ($\partial U/\partial t = 0$) means that all short-term modulation effects (with periods shorter than one solar rotation) are neglected [43].

When $S_P \approx 0$, the solution of Eq. (17) in terms of U_P, assuming that V is constants, $k = k_0 P^a r^b$, $(b > 1)$, and $U_P \to A_c P^{-\gamma}$ (put A_c = constant) at $r \to \infty$, has a form:

$$U_P = A_c P^{-\gamma} \left(1 - \frac{a}{3\,(1-b)} \frac{Vr}{k_0 P^a r^b}\right)^{\frac{1+(\gamma-1)/3}{-a/3}} . \tag{18}$$

Due to the complexity of heliospheric processes the available physical models do not give detailed description of the cosmic fluxes and their dynamics [48–50]. The task concerning GCR modulation during solar cycle belongs to the class of unsolved problems in mathematical physics and has not been unambiguously solved theoretically because of insufficient knowledge of the properties of the modulation medium (e.g., the dependence of the diffusion coefficient on helio-coordinates and on the magnetic rigidity of particles), an uncertainty of boundary conditions (e.g., GCR energy spectra at the modulation region boundary), the time variations of these characteristics throughout 11- and 22-year solar cycles [51, 52]. At the same time the actual requirements of simulating radiation environment in space impose the need to construct dynamic models of GCR modulation that would allow the radiation environment to be predicted; therefore, it is necessary to develop empirical and semi-empirical modulation models for GCR spectra [51, 52]. Semi-empirical models involve assumptions and approximations to simplify calculations and to yield a result in accord with observations. Some semi-empirical models in cosmic-ray physics are based on the Fisk and Axford' approximate solutions (see Eq. (15) and Eq. (18)) [39]. Equation (15) gives the basis of the models derived from different research groups [53, 54] and Eq. (18) is related to widely used semi-empirical Nymmiks' model [51, 55, 56]

$$D_i(P,t) = D_{i0}(P) \left(\frac{P + P_0(t)}{P}\right)^{-\Delta_i(P,\,\mathrm{sgn}z,\,P_0,\,t)} , \tag{19}$$

where $D_{i0}(P) = D_{(i)_{\rm LIS}}(P)$ are the unmodulated GCR spectra with respect to the rigidity P of i-specie of particles, $P_0 = P_0(W(\nabla T(n, P, t)))$ is the effective modulation potential which depends on the solar activity level (n is the solar cycle number), time t, and rigidity P through the lag time ∇T; $W(\nabla T(n, P, t))$ is the smoothed monthly mean sunspot number corresponding to the time $t - \nabla T$ [55, 56]. Δ_i $(P, \mathrm{sgn}z, P_0, t)$ is a dimensionless parameter that depends on the particle rigidity P, particle charge sign z, P_0 and t [55]; Δ_i describes the charge sign-dependent effects. The semi-empirical model (19) takes into account effects, caused by the 22-year dynamics of the large-scale magnetic field [50].

The cosmic-ray intensity $D(P)$, as a function of particle rigidity P, is related to the differential number density, U_P, by [32]

$$D(P) = \frac{vU_P}{4\pi} . \tag{20}$$

Then,

$$\frac{D(P)}{D_0(P)} = \frac{U_P}{U_{0P}}\,, \qquad U_{0P} = A_c P^{-\gamma}\,. \tag{21}$$

Equation (19) follows the same form as Eq. (18) when we assume that in Eq. (18) $a = 1$ and $r = 1$ AU, and take into account Eq. (21).

Kuznetsovs' [54] and Nymmiks' [51, 55, 56] semi-empirical models calculate fluxes of GCR particles as a function of solar activity (sunspot number). In these models the main parameter is the modulation potential. The semi-empirical models sets a directly proportional relationship between the modulation potential and the sunspot number $W(t - \nabla t)$, taking into account the different time delay ∇t in odd and even solar cycles [54, 56]. Kuznetsovs' [54] and Nymmiks' [51, 55, 56] models, derived from observations on statistically precise data, are based on an approximate solution to the transport equation given by Fisk and Axford [39]. These models describe the GCR spectra outside the Earth's magnetosphere at a distance of 1 AU.

Usually, the existing empirical and semi-empirical models relate the long-term (11- and 22-year) variations in the galactic cosmic-ray intensity to the observed sunspot numbers. The further development of this type of models is related to finding functional dependencies between the model parameters and different heliospheric and solar variables. The study of the dependencies between variations of solar-heliospheric parameters and long-term CR modulation and their incorporation into the existing semi-empirical models would lead to better compliance between the predicted and the observed galactic intensities during the solar cycle.

The experiments related to the measurements of the GCR primary flux contribute significantly to understanding the global aspects of modulation. They are also an opportunity to improve the existing empirical and semi-empirical models. The data provided by balloon flights (BESS, LEAP, CAPRICE, IMAX) and satellite experiments (PAMELA, AMS) enable the development of more accurate empirical and semi-empirical models. Useful cosmic-ray database (CRDB) can be found here: https://lpsc.in2p3.fr/crdb/.

4 Conclusion

Variations in primary cosmic-ray intensity are result of deflection of the galactic cosmic rays by the time-varying solar magnetic field that is convected away from the Sun by the solar wind [57]. The GCR intensity at Earth varies with time in response to solar activity, indicated by the sunspot number, with a period of ≈ 11 year. The amplitude of the variation increases with decreasing CR energy [57].

Sunspots do not cause modulation. As noted in Ref. [57] sunspots are the consequence of the diminution of the light that is emitted by the Sun by strong, localized magnetic fields, and they provide an independent record of the magnetic activity of the Sun. They are rather an indication of the time variation of other solar activity parameters such as the solar and heliospheric magnetic fields, solar wind velocity, turbulence in the HMF, and episodic disturbances in the heliosphere caused by solar flares, coronal mass ejections, corotating interaction regions, merged interactions regions, and globally merged interaction regions [57, 58]. Cabbalero-Lopez *et al.* [57] note that the aim of cosmic-ray modulation studies is to determine quantitatively how these variables create the propagation conditions that explain the cosmic-ray modulation. In Ref. [57] is analyzed the long-term modulation due to solar and heliospheric variations and is found that long-term cosmic-ray variations are better correlated with the magnitude of the heliospheric magnetic field (HMF) than the sunspot number, solar wind speed, and tilt angle of the HMF. It is also found that the force field approximation is unable to take into account the effects of three-dimensional particle transport, showing a poor correlation with the perpendicular mean free path [57].

Current CR modulation studies [57, 59, 60] describe in increasing details the long-term variations in cosmic-ray intensity in terms of variations of solar-heliospheric parameters. These studies are a contribution to the future development of more accurate semi-empirical models, describing and predicting the GCR spectra during the solar cycle, not only based on the sunspot number, but also on other solar activity parameters. Our group is working on the CRSA model that has the potential to be developed as such a type of semi-empirical model [61].

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