Bulg. J. Phys. **44** (2017) 407–416

Parameter Free Predictions within the Proxy-SU(3) Model

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Received *31 October 2017*

Abstract. Using a new approximate analytic parameter-free proxy-SU(3) scheme, we make predictions of shape observables for deformed nuclei, namely β and γ deformation variables, and compare them with empirical data and with predictions by relativistic and non-relativistic mean-field theories. Furthermore, analytic expressions are derived for $B(E2)$ ratios within the proxy-SU(3) model, free of any free parameters, and/or scaling factors. The predicted $B(E2)$ ratios are in good agreement with the experimental data for deformed rare earth nuclides.

PACS codes: 21.60.Fw, 21.60.Ev, 21.60.Cs

1 Intoduction

The proxy-SU(3) model has been recently introduced in Refs. [1, 2]. The approximations used in this scheme have been discussed and justified through a Nilsson calculation in Ref. [1], while in Ref. [2] the way to predict the β and γ deformation parameters for any nucleus, using as input only the proton number Z and the neutron number N of the nucleus, as well as the quantum numbers λ and μ appearing in the SU(3) irreducible representation (irreps) characterizing this nucleus within the proxy-SU(3) scheme, has been described in detail.

In Section 2 of the present paper we carry out in the rare earth region a detailed comparison of the proxy-SU(3) predictions to detailed results obtained with the

D1S Gogny interaction, tabulated in Ref. [3], while in Section 3 we calculate $B(E2)$ ratios within ground state bands and γ_1 bands of some deformed rare earth nuclei and compare them to the existing data [4]. In both sections, no free parameters are used.

2 Predictions for the Deformation Parameters

2.1 Connection between deformation variables and SU(3) quantum numbers

A connection between the collective variables β and γ of the collective model [5] and the quantum numbers λ and μ characterizing the irreducible representation (λ, μ) of SU(3) [6, 7] has long been established [8, 9], based on the fact that the invariant quantities of the two theories should possess the same values.

The relevant equation for β reads [8, 9]

$$\beta^2 = \frac{4\pi}{5} \frac{1}{(A\bar{r^2})^2}(\lambda^2 + \lambda\mu + \mu^2 + 3\lambda + 3\mu + 3), \tag{1}$$

where A is the mass number of the nucleus and $\bar{r^2}$ is related to the dimensionless mean square radius [10], $\sqrt{\bar{r^2}} = r_0 A^{1/6}$. The constant r_0 is determined from a fit over a wide range of nuclei [11, 12]. We use the value in Ref. [8], $r_0 = 0.87$, in agreement to Ref. [12]. The quantity in Eq. (1) is proportional to the second order Casimir operator of SU(3) [13],

$$C_2(\lambda, \mu) = \frac{2}{3}(\lambda^2 + \lambda\mu + \mu^2 + 3\lambda + 3\mu). \tag{2}$$

The relevant equation for γ reads [8, 9]

$$\gamma = \arctan\left(\frac{\sqrt{3}(\mu + 1)}{2\lambda + \mu + 3}\right). \tag{3}$$

2.2 Numerical results

In Figure 1 (Figure 2) results for the collective variable β (γ) are shown, calculated from Eq. (1) [Eq. (3)] and rescaled in the case of β as described in detail in Ref. [2]. Experimental results obtained from Ref. [15] are also shown for comparison, as in Ref. [2]. Furthermore, comparison to the detailed results provided by the D1S Gogny force, tabulated in Ref. [3], is made. By "Gogny D1S mean" we label the mean ground state β (γ) deformation [entry 11 (12) in the tables of [3]], while the error bars correspond to the variance of the ground state β (γ) deformation [entry 13 (14) in [3]]. By "Gogny D1S min." we label the β (γ) deformation at the HFB energy minimum [entry 4 (5) in [3]].

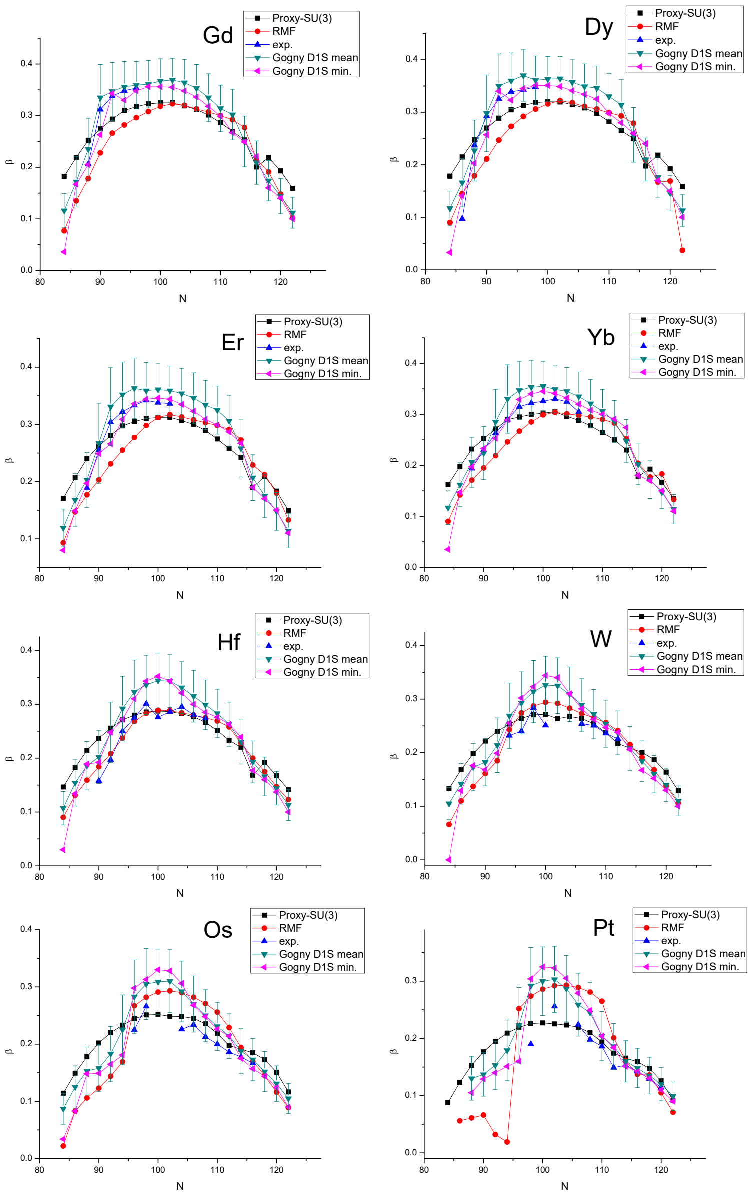


Figure 1. (color online) Proxy SU(3) predictions for Gd-Pt isotopes for β, obtained from Eq. (1), as described in detail in Ref. [2], compared with results by the D1S-Gogny interaction (D1S-Gogny) [3] and by relativistic mean field theory (RMF) [14], as well as with empirical values (exp.) [15]. See subsection 2.2 for further discussion.

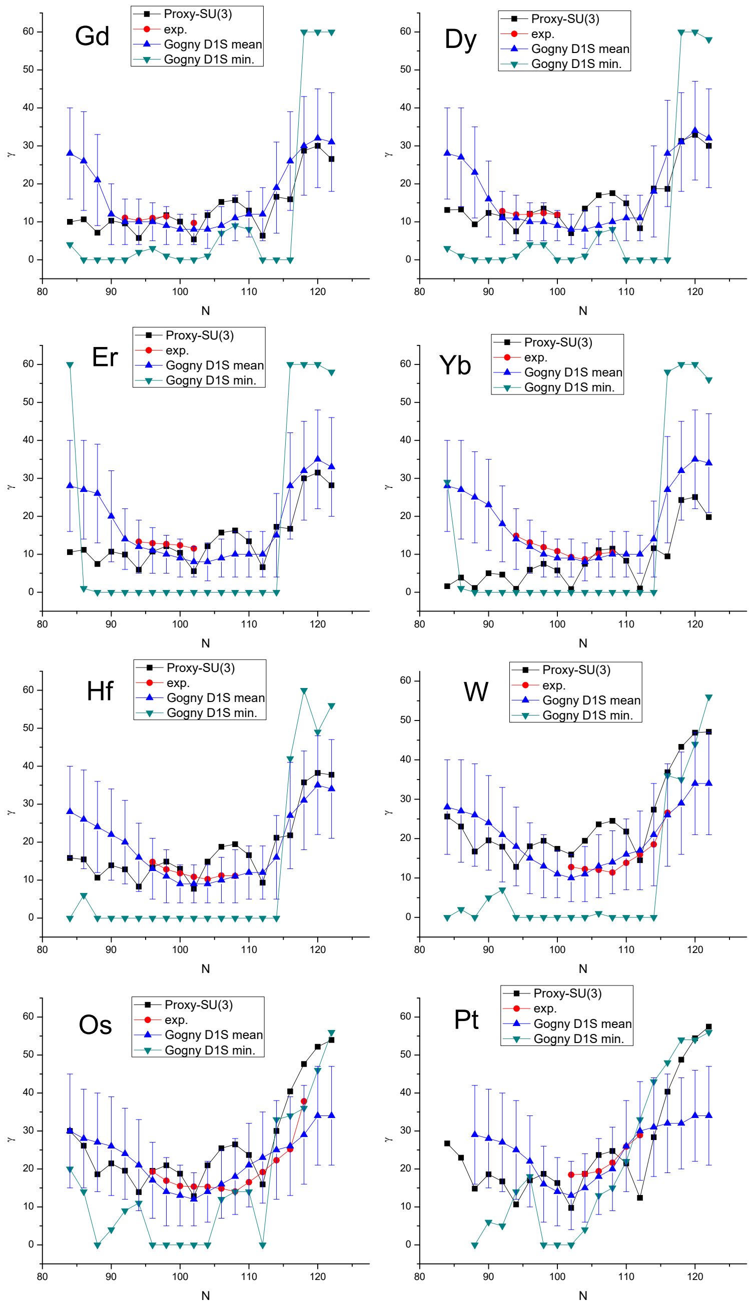


Figure 2. (color online) Same as Figure 1, but for γ, derived from Eq. (3). See subsection 2.2 for further discussion.

In the case of β, predictions obtained with relativistic mean field theory (RMF) [14] are also shown.

In the case of β we note that the HFB minimum lies always within the error bars of the D1S Gogny mean g.s. deformation (except for $N = 84$), while in the case of γ we see that the HFB minimum lies well below the error bars of the D1S Gogny mean g.s. deformation for most of the N values, but jumps suddenly to very high values, close to 60 degrees, at or near $N = 116$.

In Figure 1 the proxy-SU(3) predictions for β lie within the error bars of the D1S Gogny mean g.s. deformation, with the following few exceptions: a) The first ($N = 84$) point in Gd-Hf, b) the last two ($N = 120, 122$) points in Gd and Dy, as well as the last point ($N = 122$) in Er, c) a few isolated cases, like the $N = 110$ point in Er, the $N = 102$ point in W and Os, and the $N = 100, 102$ points in Pt. We stress, however, that proxy-SU(3) is only valid for deformed nuclei and therefore some of these differences [items a) and b)] may not be meaningful.

Similar observations can be made for γ in Figure 2, where the exceptions occur in: a) The first three points ($N = 84, 86, 88$) in Gd-Yb, b) the last three ($N = 118, 120, 122$) points in W-Pt, which agree with the HFB minimum rather than with the mean g.s. deformation, c) a few isolated cases, like some of the $N = 106$ in Gd, Dy, Er, Hf, W, and several points in Yb.

2.3 Discussion

The above observations can be summarized as follows:

1) While the β deformation at the HFB energy minumum remains always close to the mean ground state β deformation, the behavior of the γ deformation is strikingly different. In most of the region the γ deformation at the HFB energy minumum remains close to zero, but it suddenly jumps to values close to 60 degrees near the end of the shell ($N = 116$-122). This jump is sudden in Gd-Hf, while it becomes more gradual in W, Os, Pt.
2) In the beginning of the region we see some failures of proxy-SU(3) at $N = 84, 86, 88$ in Gd-Hf. These failures are expected, since these nuclei are not well deformed, as known from their $R_{4/2}$ ratios.
3) In most of the region, the proxy-SU(3) predictions for both β and γ are in good agreement with the D1S Gogny mean g.s. deformations.
4) The agreement of the proxy-SU(3) predictions with the D1S Gogny mean g.s. deformations remains good up to the end of the shell for β, while for γ in W, Os, Pt it is observed that the proxy-SU(3) predictions for γ jump at the end of the shell from close agreement to the D1S Gogny mean g.s. deformations to close agreement with γ at the HFB energy minimum, i.e., close to 60 degrees.

3 B(E2) Ratios

As discussed in Appendix A, $B(E2)$s within the proxy-SU(3) model are proportional to the square of the relevant reduced matrix element of the quadrupole operator Q. If ratios of $B(E2)$s within the same nucleus and within the same irreducible representation are considered, only the relevant SU(3)→SO(3) coupling coefficients remain, while all other factors cancel out, leading to

$$\frac{B(E2;L_i \to L_f)}{B(E2;2_g \to 0_g)} = 5\frac{2L_f+1}{2L_i+1}\frac{(\langle(\lambda,\mu)K_iL_i;(1,1)2||(\lambda,\mu)K_fL_f\rangle)^2}{(\langle(\lambda,\mu)02;(1,1)2||(\lambda,\mu)00\rangle)^2}, \quad (4)$$

where normalization to the $B(E2)$ connecting the first excited 2^+ state to the 0^+ ground state of even-even nuclei is made. The needed SU(3)→SO(3) coupling coefficients are readily obtained from the SU3CGVCS code [16], as described in Appendix A.

It should be noticed that the ratios given by Eq. (4) are completely free of any free parameters and/or scaling factors.

3.1 Numerical results

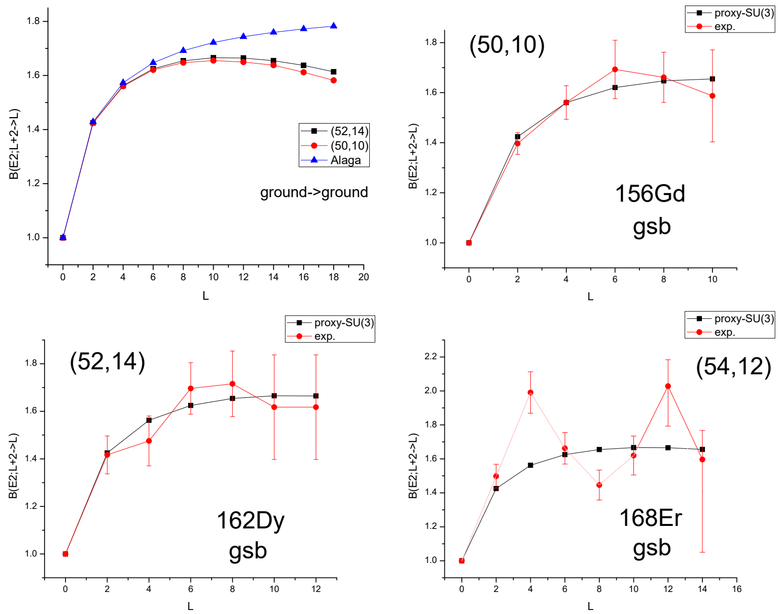
Calculations have been performed for the proxy-SU(3) irreps (54,12), (52,14), and (50,10). The irrep (54,12) accommodates ^{168}Er, for which complete spectroscopy has been performed [17], and ^{160}Gd, for which little data on $B(E2)$s exist [4]. The irrep (52,14) accommodates ^{162}Dy, for which complete spectroscopy has been performed [18], and ^{166}Er, for which rich data exist [4]. It also accommodates ^{172}Er, for which little data on $B(E2)$s exist [4]. The irrep (50,10) accommodates ^{156}Gd, which has been cited as the textbook example of the bosonic SU(3) in the IBM-1 framework [13]. The Alaga values [19], derived from the relevant Clebsch-Gordan coefficients alone, are also given for comparison.

3.2 Comparisons to experimental data for specific nuclei

$B(E2)$s within the ground state band are shown in Figure 3. Agreement between the proxy-SU(3) predictions and the data is excellent in the cases of ^{156}Gd, ^{162}Dy, and ^{166}Er, while in ^{168}Er three points are missed. It appears that nuclear stretching [20] has been properly taken into account.

In Figure 4 three pairs of nuclei, each pair accommodated within a single proxy-SU(3) irrep, are shown. These are the only pairs for which adequate data exist [4] in the region of 50-82 protons and 82-126 neutrons. Agreement within the experimental errors is seen in almost all cases.

Proxy-SU(3) predictions for $B(E2)$s within the γ_1 band, with $\Delta L = -2$ (increasing with L) and $\Delta L = -1$ (decreasing with L), are shown in Figure 5,



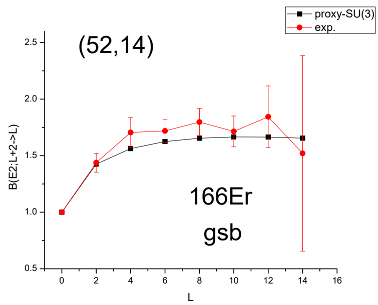


Figure 3. (color online) B(E2)s within the ground state band are shown for the indicated proxy-SU(3) irreps and for four nuclei, with data taken from [4]. All values are normalized to $B(E2; 2_1^+ \to 0_1^+)$. Results for (54,12) are not shown in the upper left panel, because for this band they are very similar to those of (52,14). See subsection 3.2 for further discussion.

and are compared to the data for nuclei for which sufficient data exist [4]. The distinction between increasing $B(E2)$s with $\Delta L = -2$ and decreasing $B(E2)$s with $\Delta L = -1$ is seen clearly in the data.

3.3 Discussion

The main findings of the present section can be summarized as follows.

Analytic expressions for $B(E2)$ ratios for heavy deformed nuclei providing numerical results in good agreement with experiment are derived within the proxy-SU(3) scheme without using any free parameters and/or scaling factors. The derivation, described in Appendix A, is exact. The only quantities appearing in the final formula are the relevant SU(3)→SO(3) coupling coefficients, for which computer codes are readily available [16, 21].

Concerning further work, spectra of heavy deformed nuclei will be considered

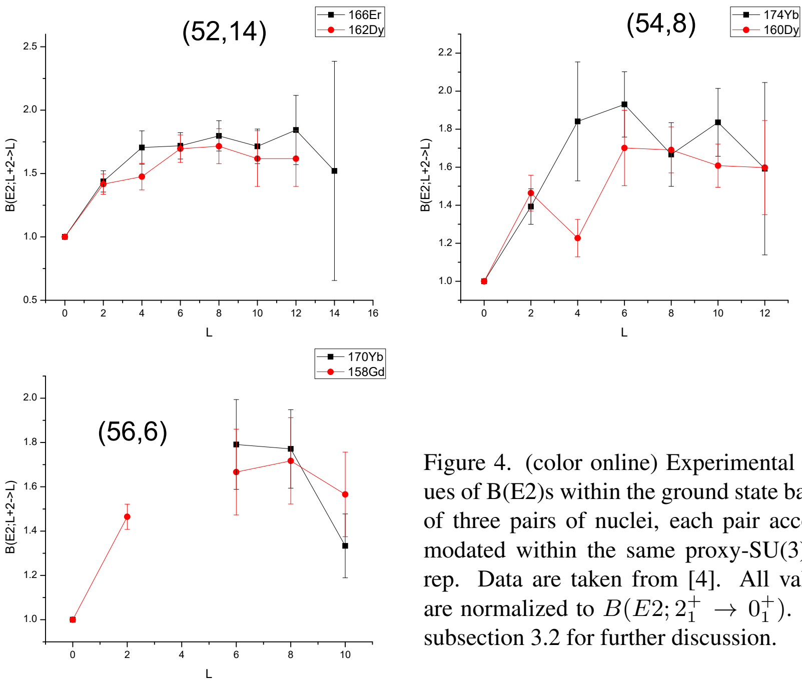


Figure 4. (color online) Experimental values of B(E2)s within the ground state bands of three pairs of nuclei, each pair accommodated within the same proxy-SU(3) irrep. Data are taken from [4]. All values are normalized to $B(E2; 2_1^+ \to 0_1^+)$. See subsection 3.2 for further discussion.

within the proxy-SU(3) scheme, involving three- and/or four-body terms in order to break the degeneracy between the ground state and γ_1 bands [22–24]. Furthermore, $B(M1)$ transition rates can be considered along the proxy-SU(3) path, using the techniques already developed [25] in the framework of the pseudo-SU(3) scheme.

Acknowledgements

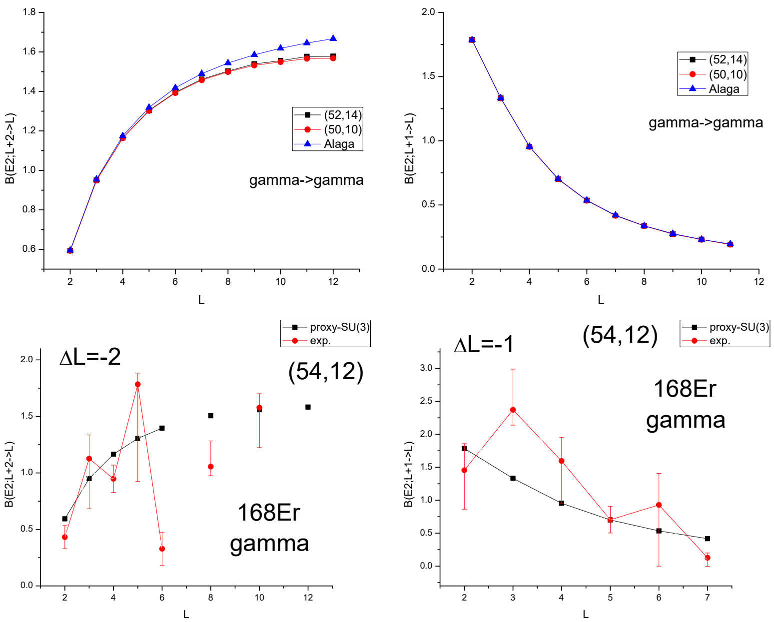
Work partly supported by the Bulgarian National Science Fund (BNSF) under Contract No. DFNI-E02/6, by the US DOE under Grant No. DE-FG02- 91ER-40609, and by the MSU-FRIB laboratory, by the Max Planck Partner group, TUBA-GEBIP, and by the Istanbul University Scientific Research Project No. 54135.

Appendix A. Formulae used for $B(E2)$s

In most of the earlier work, effective charges

$$e_\pi = e + e_{eff}, \qquad e_\nu = e_{eff}, \tag{5}$$

have been used, where the effective charge e_{eff} is usually fixed so that the calculated $B(E2)$ transition rate for the $2_1^+ \to 0_1^+$ transition reproduces the experi-



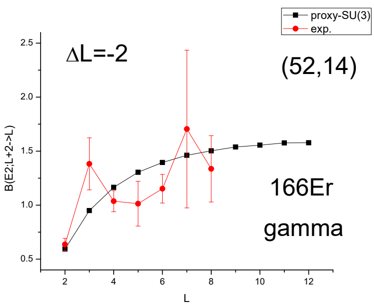


Figure 5. (color online) B(E2)s within the γ_1 band are shown for the indicated proxy-SU(3) irreps and for two nuclei, for which sufficient data exist [4]. All values are normalized to $B(E2; 2_1^+ \to 0_1^+)$. See subsection 3.2 for further discussion.

mental value [25]. In the present approach we make the choice $e_{eff} = 0$, which leads to $e_\pi = e$ and $e_\nu = 0$.

The needed matrix elements of the relevant quadrupole operators, Q^π and Q^ν for protons and neutrons respectively, are given in detail in Appendix D of Ref. [23], with SU(3)$\to$SO(3) coupling coefficients [16, 21, 26, 27], as well as 9-(λ, μ) coefficients [21, 26, 28] appearing in the relevant expressions. Codes for calculating these coefficients are readily available, given in the references just cited. With $e_{eff} = 0$ one sees that only the matrix elements of Q^π are needed. Furthermore, if we use ratios of $B(E2)$ transition rates within a given nucleus, the 9-(λ, μ) coefficients will cancel out and the only nontrivial term remaining in the B(E2) ratios will be the ratio of the relevant SU(3)$\to$SO(3) coupling coefficients, which remarkably involve only the highest weight (λ, μ) irrep characterizing the whole nucleus, while they are independent of the (λ_π, μ_π) and (λ_ν, μ_ν) irreps characterizing the protons and the neutrons separately.

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