

# Conformally Flat Tilted Cosmological Models in General Relativity

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**Abstract.** Conformally flat tilted plane symmetric cosmological models of bulk viscous fluid with heat conduction are investigated. It is assumed that a relation between metric potentials  $A = B^n$ . The physical and geometrical aspects of the model are also discussed.

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## 1 Introduction

The state in which radiation concentrates around the star is described by general relativity. Klein [1] has investigated an approximate solution to Einstein field equation in spherical symmetry for a distribution of diffused radiation. Singh and Abdussattar [2]; Roy and Bali [3] have obtained a static spherically and cylindrically symmetric solution to Einstein field equation for perfect fluid filled with disordered radiation. Teixeira, Wolk, Som [4] have investigated a model filled with source free disordered distribution of electromagnetic radiation in general relativity. Non-static plane symmetric cosmological model filled with disordered radiation has been obtained by Roy and Singh [5]. These are orthogonal universes in which the matter moves orthogonally to the hyper surface of homogeneity. Considerable interest has been focused in investigating spatially homogenous and anisotropic universes in which the matter does not move orthogonally to the hyper surface of homogeneity in recent years. These are called tilted universes. King and Ellis [6]; Ellis and King [7]; Collins and Ellis [8] have studied the general dynamics of tilted cosmological models.

Dunn and Tupper [9] have been studied tilted Bianchi type I cosmological model for perfect fluid. Cen *et al.* [10] has discussed the tilted cosmology with cold dark matter. Many other researchers like Matravers *et al.* [11], Hewitt *et al.* [12,13], Horwood *et al.* [14], Bali and Sharma [15], Apostolopoulos [16] have studied different aspects of tilted cosmological models. Coley and Tupper

[17,18], Bradley and Sviestiny [19], Banerjee *et al.* [20], Roy and Banerjee [21], Roy and Prasad [22] have investigated the cosmological models with heat flow. Bianchi type V tilted cosmological models in the Scale-covariant theory derived by Beesham [23]. Bali and Meena [24] have investigated two tilted cosmological models filled with disordered radiation of perfect fluid and heat flow.

Cosmological models play an important role in understanding some essential features of the Universe such as formation of galaxies during the early stages of evolution. In homogeneous cosmological models of plane symmetric space-time weve been considered by Taub [25,26] and Tomimura [27]. It is therefore interesting to carry out the detailed studies of gravitational fields which are described by plane symmetric space-time. In general theory of relativity, a number of authors have studied cosmological models with bulk viscosity – Pavon [28], Maartens [29], Pradhan *et al.* [30]. It forms the motivation for us to study the cosmological models with viscous fluid.

Recently, Bali and Meena [31], have investigated two conformally flat tilted Bianchi type V cosmological models filled with perfect fluid and conduction. Bali and Sharma [32] have investigated tilted Bianchi type I cosmological model for perfect fluid distribution in the presence of magnetic field. Pradhan and Rai [33,34] have obtained tilted Bianchi type I and conformally flat tilted Bianchi type V cosmological models filled with disordered radiation in the presence of a bulk viscous fluid and heat flow. Pawar *et al.* [35] has studied tilted plane symmetric cosmological models with heat conduction and disordered radiation.

In this paper we intend to construct conformally flat tilted plane symmetric cosmological model with bulk viscosity. To get realistic tilted model, here we assumed that a relation between metric potential  $A = B^3$ . The physical and geometrical aspects of the model are also discussed. This paper is organized as follows. The metric and field equations are presented in Section 2. Section 3 deals with solution of field equation a in the presence of bulk viscous fluid. In Section 4 we discussed some physical and geometrical properties of the models. Sections 5 and 6 are devoted to the solution of particular models and their discussion.

## 2 Metric and Field Equations

We consider the plane symmetric metric in the form

$$ds^2 = - dt^2 + A^2 (dx^2 + dy^2) + B^2 dz^2, \quad (1)$$

where  $A$  and  $B$  are the function of time only.

The field equations of Einstein in general relativity is

$$G_{ij} = -8\pi T_{ij}, \quad (2)$$

where  $G_{ij}$  is the Einstein tensor and we choose the units such that  $c = 1, G = 1$ .

The energy momentum tensor is given by

$$T_i^j = (\rho + \bar{p}) v_i v^j + \bar{p} g_i^j + q_i v^j + v_i q^j \quad (3)$$

and

$$\bar{p} = p - \xi v^i_{;i}, \quad (4)$$

$$g_{ij} v^i v^j = -1, \quad (5)$$

$$q_i q^j > 0, \quad (6)$$

$$q_i v^i = 0, \quad (7)$$

where  $\rho$  is the density,  $p$  is the pressure,  $\bar{p}$  is the effective pressure,  $\xi$  is the bulk viscous coefficient and  $q_i$  is the heat conduction vector orthogonal to  $v^i$ .

The fluid flow vector  $v^i$  has the components  $\left(0, 0, \frac{\sin h\alpha}{B}, \cos h\alpha\right)$  satisfying equation (5),  $\alpha$  being the tilt angle.

The field equations (2) for metric (1) reduce to

$$\frac{A_{44}}{A} + \frac{A_4 B_4}{AB} + \frac{B_{44}}{B} = -8\pi \bar{p}, \quad (8)$$

$$\frac{A_4^2}{A^2} + \frac{2A_{44}}{A} = -8\pi \left[ (\rho + \bar{p}) \sin h^2 \alpha + \bar{p} + 2q_3 \frac{\sin h\alpha}{B} \right], \quad (9)$$

$$\frac{A_4^2}{A^2} + \frac{2A_4 B_4}{AB} = -8\pi \left[ -(\rho + \bar{p}) \cos h^2 \alpha + \bar{p} - 2q_3 \frac{\sin h\alpha}{B} \right], \quad (10)$$

$$-8\pi \left[ (\rho + \bar{p}) B \sin h\alpha \cos h\alpha + q_3 \cos h\alpha - q_3 \frac{\sin h^2 \alpha}{\cos h\alpha} \right] = 0. \quad (11)$$

Here the index 4 after a field variable denotes the differentiation with respect to time  $t$ .

### 3 Solutions of the Field Equations

The set of four field equations (8) – (11) being highly non-linear containing seven unknowns ( $A, B, \alpha, \rho, p, \xi$ , and  $q_3$ ). So to obtain a determinate solution, we assume that the space-time is conformally flat, which gives

$$C_{1212} = \frac{1}{3} \left[ A^2 A_4^2 - A^3 A_{44} - A^3 A_4 \frac{B_4}{B} + A^4 \frac{B_{44}}{B} \right], \quad (12)$$

and secondly we assume,

$$A = B^n, \quad (13)$$

where  $n$  is constant.

Equations (12) and (13) lead to

$$B^3 B_{44} + (n-1)B^2 B_4^2 = 0. \quad (14)$$

From Eqs. (9), (10) and (13), we have

$$(n^2 - 2n) \frac{B_4^2}{B^2} + n \frac{B_{44}}{B} = -4\pi \left[ (\rho + \bar{p}) \cos 2h\alpha + 4q_3 \frac{\sin h\alpha}{B} \right] \quad (15)$$

and

$$n \frac{B_{44}}{B} + 2n^2 \frac{B_4^2}{B^2} = 4\pi(\rho - \bar{p}). \quad (16)$$

Now, from Eqs. (8) and (16), we get

$$n^2 \frac{B_4^2}{B^2} - \frac{B_{44}}{B} = 4\pi(\rho + \bar{p}). \quad (17)$$

From Eqs. (11) and (13), we have

$$-16\pi q_3 \sin h\alpha = 4\pi(\rho + \bar{p})B \sin 2h\alpha \tan 2h\alpha. \quad (18)$$

From equations (15) and (18), we obtain

$$(n^2 - 2n) \frac{B_4^2}{B^2} + n \frac{B_{44}}{B} = \frac{-4\pi(\rho + \bar{p})}{\cos 2h\alpha}. \quad (19)$$

Equation (14) can be rewritten as

$$\frac{B_{44}}{B_4} + (n-1) \frac{B_4}{B} = 0. \quad (20)$$

We obtained the solution of Eq. (20) as

$$B = n^{\frac{1}{n}} (at + b)^{\frac{1}{n}}, \quad (21)$$

where  $a, b$  are constants of integration.

$$A^2 = n^2 (at + b)^2, \quad (22)$$

$$B^2 = n^{\frac{2}{n}} (at + b)^{\frac{2}{n}}. \quad (23)$$

Hence the line element (1) reduces to

$$ds^2 = -dt^2 + n^n (at + b)^2 (dx^2 + dy^2) + n^{\frac{2}{n}} (at + b)^{\frac{2}{n}} dz^2. \quad (24)$$

Using the co-ordinate transformation

$$ds^2 = -\frac{dT^2}{2} + n^2 T^2 (dX^2 + dY^2) + n^{\frac{2}{n}} T^{\frac{2}{n}} dz^2. \quad (25)$$

#### 4 Some Physical and Geometrical Properties

The effective pressure and density for model (25) are given by

$$8\pi\bar{p} = 8\pi(p - \xi\theta) = -\frac{2n^2a}{T}, \quad (26)$$

$$8\pi\rho = 6\frac{n^2a}{T}. \quad (27)$$

The scalar of expansion ( $\theta$ ), the flow vector  $v^i$  for model (25) are given by

$$\theta = \left(\frac{2}{n} + 1\right)\frac{a}{T}k. \quad (28)$$

The tilt angle  $\alpha$  is given by

$$\cos h^2\alpha = k^2, \quad (29)$$

$$\sin h^2\alpha = K^2, \quad (30)$$

where  $k$  and  $K$  are constants given by

$$k^2 = \frac{(n-1)}{(n-2)}, \quad (31)$$

$$K^2 = \frac{1}{(n-2)}. \quad (32)$$

Let us assume the following *ad hoc* law [29, 36].

$$\xi(t) = \xi_0 p^m, \quad (33)$$

where  $\xi_0$  and  $m$  are real constants, if  $m = 1$ , Eq. (33) may correspond to a radioactive fluid [37].

##### 4.1 Case I

When  $m = 0$ , equation (33) reduces to  $\xi = \xi_0 = \text{const}$  and hence Eqs. (26) with the use of (28) are given by

$$p = \xi_0 \left(\frac{2}{n} + 1\right)\frac{a}{T}k - \frac{2n^2a}{8\pi T}. \quad (34)$$

##### 4.2 Case II

When  $m = 1$ , equation (33) reduces to  $\xi = \xi_0\rho$  and hence Eqs. (26) with the use of (28) are given by

$$p = \frac{1}{8\pi T} \left[ -2n^2a + \xi_0 6n^2a \left(\frac{2}{n} + 1\right)\frac{a}{T}k \right]. \quad (35)$$

We observe that, when  $T = 0$  the pressure  $p$  and density  $\rho$  are infinite, time increases, pressure and density decreases, finally  $T \rightarrow \infty$ ,  $p$  and  $\rho$  both are zero.

The weak and strong energy conditions, in Case I,

$$\rho + p = \frac{n^2 a}{2\pi T} + \xi_0 \left( \frac{2}{n} + 1 \right) \frac{a}{T} k, \quad (36)$$

$$\rho - p = \frac{n^2 a}{\pi T} + \xi_0 \left( \frac{2}{n} + 1 \right) \frac{a}{T} k, \quad (37)$$

$$\rho + 3p = 3\xi_0 \left( \frac{2}{n} + 1 \right) \frac{a}{T} k, \quad (38)$$

$$\rho - p = \frac{3n^2 a}{2\pi T} + 3\xi_0 \left( \frac{2}{n} + 1 \right) \frac{a}{T} k. \quad (39)$$

In Case II, we have

$$\rho + p = \frac{1}{8\pi T} \left[ 4n^2 a + 6\xi_0 n^2 \left( \frac{2}{n} + 1 \right) \frac{a^2}{T} k \right], \quad (40)$$

$$\rho - p = \frac{1}{8\pi T} \left[ 8n^2 a + 6\xi_0 \left( \frac{2}{n} + 1 \right) \frac{a^2}{T} k \right], \quad (41)$$

$$\rho + 3p = \frac{18\xi_0 n^2 \left( \frac{2}{n} + 1 \right) a k}{8\pi T^2}, \quad (42)$$

$$\rho - 3p = \frac{3}{8\pi T} \left[ 4n^2 a + 6\xi_0 n^2 \left( \frac{2}{n} + 1 \right) \frac{a^2}{T} k \right]. \quad (43)$$

The reality conditions  $\rho \geq 0$ ,  $p \geq 0$  and  $\rho - 3p \geq 0$  imposes further constraint on both of the above cases.

The flow vector  $v^i$  and heat conduction  $q^i$  for model (25) are given by

$$v^3 = \frac{K}{nT}, \quad (44)$$

$$v^4 = k, \quad (45)$$

$$q^3 = \frac{-naKk^2}{2\pi(k^2 + K^2t)T^2}, \quad (46)$$

$$q^4 = \frac{aK^2k}{\pi(k^2 + K^2)T^3}. \quad (47)$$

The rates of expansion  $H_i$  in the direction of  $x$ ,  $y$ ,  $z$  axis are given by

$$H_1 = H_2 = \frac{2a}{nT}, \quad (48)$$

$$H_3 = \frac{2a}{T}. \quad (49)$$

The non-vanishing components of shear tensor ( $\sigma_{ij}$ ) are given by

$$\sigma_{11} = \sigma_{22} = n^{\frac{2}{n}} T^{\frac{2}{n}} ak \left[ \frac{3 - n \left( \frac{2}{n} + 1 \right)}{3nT} \right], \quad (50)$$

$$\sigma_{33} = n^2 T ak (1 + K^2) \left( 1 - \frac{1}{3} \left( \frac{2}{n} + 1 \right) \right), \quad (51)$$

$$\sigma_{44} = \frac{ak}{T} \left[ K^2 - \frac{1}{3} \left( \frac{1}{2} k^2 - 1 \right) \left( \frac{2}{n} + 1 \right) \right], \quad (52)$$

$$\sigma_{34} = \frac{1}{2} naK \left[ \frac{k^2}{\sigma} \left[ 2 \left( \frac{2}{n} + 1 \right) - 3 \right] - (1 + K^2) \right]. \quad (53)$$

All rotation tensors ( $\omega_{ij}$ ) vanish.

The study results into an expanding shearing and non-rotating universe. When  $T \rightarrow 0$ , density and pressure are both infinite. The universe starts expanding with big bang at  $T = 0$ . The expansion rate decreases as time proceeds ahead and in the event, when  $T \rightarrow \infty$ , the expansion stops where pressure and density become zero.

$$v^3 = 0 \quad \text{and} \quad v^4 = k.$$

Since  $\lim_{T \rightarrow \infty} \left( \frac{\sigma}{\theta} \right) \neq 0$  to the model (25) is devoid of isotropy for large value of  $T$ . There is a real physical singularity in model when  $T \rightarrow 0$ .

## 5 Particular Model

For sensible tilted model, we assume the condition in metric potential as  $A = B^3$ .

By using the condition, the geometry of the space-time (25) reduces to the form

$$ds^2 = -\frac{dT^2}{2} + 9T^2(dX^2 + dY^2) + 3^{\frac{2}{3}} T^{\frac{2}{3}} dZ^2. \quad (54)$$

## 6 Some Physical and Geometrical Properties of Particular Models

The effective pressure and density for the model (54) are given by

$$8\pi\bar{p} = 8\pi(p - \xi\theta) = -\frac{18a}{T}, \quad (55)$$

$$8\pi\rho = \frac{54a}{T}. \quad (56)$$

where  $\theta$  is the scalar expansion obtained as

$$\theta = \frac{5ak_1}{3T}. \quad (57)$$

The tilt angle  $\alpha$  is given by

$$\cos h^2\alpha = k_1^2, \quad (58)$$

$$\sin h^2\alpha = K_1^2, \quad (59)$$

where  $k_1$  and  $K_1$  are constants given by

$$k_1^2 = 2, \quad (60)$$

$$K_1^2 = 1. \quad (61)$$

Thus, for given  $\xi(t)$  one can solve the system for the physical quantities.

### 6.1 Case I

When  $m = 0$  Eq. (33) reduces to  $\xi = \xi_0 = \text{const}$  and hence Eq. (55) with the use of (57) leads to

$$p = \frac{5\xi_0 a k_1}{3T} - \frac{9a}{4\pi T}. \quad (62)$$

### 6.2 Case II

When  $m = 0$  Eq. (33) reduces to  $\xi = \xi_0 \rho$  and hence Eq. (55) with the use of (57) leads to

$$p = \frac{9}{4\pi T} \left[ -a + \frac{5\xi_0 a^2 k_1}{T} \right]. \quad (63)$$

We observe that, when  $T = 0$  the pressure  $p$  and density  $\rho$  are infinite, when time increases pressure and density decreases, finally  $T \rightarrow \infty$ ,  $p$  and  $\rho$  both are zero.

The weak and strong energy condition, we have in Case I.

$$\rho + p = \frac{9a}{2\pi T} + \frac{5\xi_0 a k_1}{3T}, \quad (64)$$

$$\rho - p = \frac{9a}{\pi T} + \frac{5\xi_0 a k_1}{3T}, \quad (65)$$

$$\rho + 3p = \frac{5\xi_0 a k_1}{T}, \quad (66)$$

$$\rho - 3p = \frac{27a}{2\pi T} - \frac{5\xi_0 a k_1}{T}. \quad (67)$$

In Case II, we have

$$\rho + p = \frac{9}{4\pi T} \left[ 2a + \frac{5\xi_0 a^2 k_1}{T} \right], \quad (68)$$

$$\rho - p = \frac{1}{4\pi T} \left[ 36a + \frac{5\xi_0 a^2 k_1}{T} \right], \quad (69)$$

$$\rho + 3p = \frac{27\xi_0 a k_1}{8\pi T^2}, \quad (70)$$

$$\rho - 3p = \frac{27}{4\pi T} \left[ 2a - \frac{5\xi_0 a^2 k_1}{T} \right]. \quad (71)$$

The reality conditions  $\rho \geq 0$ ,  $p \geq 0$  and  $\rho - 3p \geq 0$  impose further constraint on both of the above cases.

The flow vector  $v^i$  and heat conduction  $q^i$  for the model (54) are given by

$$v^3 = \frac{K_1}{3T}, \quad (72)$$

$$v^4 = k_1, \quad (73)$$

$$q^3 = -\frac{3aK_1 k_1^2}{2\pi(k_1^2 + K_1^2)T^2}, \quad (74)$$

$$q^4 = \frac{aK_1^2 k_1}{\pi(k_1^2 + K_1^2)T^3}. \quad (75)$$

The rates of expansion  $H_i$  in the direction of  $x, y, z$ -axis are given by

$$H_1 = H_2 = \frac{2a}{3T}, \quad (76)$$

$$H_3 = \frac{2a}{T}. \quad (77)$$

The non-vanishing components of shear tensor ( $\sigma_{ij}$ ) are given by

$$\sigma_{11} = \sigma_{22} = \frac{-2ak_1}{3^{\frac{4}{3}}T^{\frac{1}{3}}}, \quad (78)$$

$$\sigma_{33} = 4Tak_1(1 + K_1^2), \quad (79)$$

$$\sigma_{44} = \frac{ak_1}{T} \left[ K_1^2 - \frac{5}{9} \left( \frac{k_1^2}{2} - 1 \right) \right], \quad (80)$$

$$\sigma_{34} = \frac{aK_1}{4} \left[ \frac{k_1^2}{6} - (1 + K_1^2) \right]. \quad (81)$$

All rotation tensors ( $\omega_{ij}$ ) vanish.

The study results into an expanding shearing and non-rotating universe. When  $T \rightarrow 0$ , then density and pressure are both infinite. The universe starts expanding with the big bang at  $T = 0$ . The expansion rate decreases as time proceeds ahead and in the event, when  $T \rightarrow \infty$ , the expansion stops where pressure and density become zero.

$$v^3 = 0 \quad \text{and} \quad v^4 = k_1.$$

Since  $\lim_{T \rightarrow \infty} \left(\frac{\sigma}{\theta}\right) \neq 0$  to the model (54) is devoid of isotropy for large value of  $T$ . There is a real physical singularity in model when  $T \rightarrow 0$ .

This confirms that the universe remains anisotropic through out the evolution provided  $n \neq 1$ . There is a real physical singularity in the model when  $T \rightarrow 0$ .

## 7 Conclusion

We have obtained conformally flat tilted plane symmetric cosmological models of bulk viscous fluid with heat conduction. The models are expanding, shearing and non-rotating. When  $T \rightarrow 0$ , density and pressure are both infinite. The universe starts expanding with big bang at  $T = 0$ . The expansion rate decreases as time proceeds ahead and in the event, when  $T \rightarrow \infty$ , the expansion stops where pressure and density become zero. We obtained realistic tilted model by assuming a special condition for  $n = 3$  and discussed their properties.

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