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DOES A SCALAR MESON FIELD REPRESENT AN IRROTATIONAL PERFECT FLUID IN BIMETRIC THEORY?

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Abstract. The problems of homogeneous plane symmetric perfect fluid and massive scalar field are investigated in Rosen's bimetric theory. It is shown that a macro cosmological model represented by perfect fluid distribution does not exist and only a vacuum model can be constructed whereas in case of a micro cosmological model represented by a scalar meson field exists and the model is obtained. Moreover it is shown that the massive scalar field cannot be equivalent to irrotational perfect fluid neither through the identification of the corresponding eigenvalues of their energy momentum tensors nor through the transformation as in the case of Tiwary et al and Tabensky and Taub respectively in general theory of relativity.

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1. Introduction

Rosen [1] has proposed a new theory of gravitation, which is known as bimetric theory of gravitation. It is based on a simple form of Lagrangian and has simpler mathematical structure than that of general theory of relativity. It also satisfies the covariance and equivalence principles. This theory makes use of Riemannian metric tensor g_{ij} and an associated flat space metric tensor γ_{ij} describing the properties of space-time while g_{ij} describes the gravitational potential tensor which determines the interaction between matter and gravitation. Rosen [1, 4–7], Israelit [8, 9], Karade and Dhoble [10], Karade [11], Reddy and Venkateswarlu [12], Reddy and Venkateswar Rao [13] have studied several aspects of this theory. Recently Mohanty et al [14] and Mohanty and Sahoo [15] have shown non-existence of mesonic cosmological models in Bianchi Type-I and static plane symmetric space-times respectively.

In this paper we have shown that plane symmetric cosmological perfect fluid model does not exist whereas the corresponding model can be obtained in case of scalar meson field in bimetric theory of gravitation. Further it is shown that the equivalence of massive scalar field and irrotational perfect fluid is not possible neither with the help of identification of corresponding eigenvalues of their energy momentum tensors (Tiwari, Rao and Mohanty [2]) nor by transformations (Tabensky and Taub [3]).

2. Field Equations

The field equations of the bimetric theory of gravitation formulated by Rosen [1] are

$$N_{ij} - \frac{1}{2} N g_{ij} = -8\pi k T_{ij} \tag{1}$$

where

$$N = N^i_j = \frac{1}{2}\gamma^{ab}\left(g^{hi} g_{hj}|_a\right)|_b$$

and $k = \sqrt{\frac{g}{\gamma}}$ with $g = \det g_{ij}$ and $\gamma = \det \gamma_{ij}$. Here a vertical bar | stands for covariant differentiation with respect to γ_{ij} and T_{ij} is the energy momentum tensor of matter field.

3. Perfect Fluid

We consider here the plane symmetric line element of the form

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2\, dz^2 \tag{2}$$

where A and B are functions of t only.

The background flat space-time corresponding to the metric (2) is

$$d\sigma^2 = dt^2 - dx^2 - dy^2 - dz^2\,. \tag{3}$$

The energy momentum tensor for perfect fluid is given by

$$T_{ij} = (\rho + p)U_i U_j - p g_{ij} \tag{4}$$

together with

$$g_{ij} U^i U^j = 1$$

where U^i is the four velocity vector of the fluid distribution having p and ρ as the proper pressure and energy density of the fluid respectively.

Using co-moving co-ordinate system the field equations (1) for the metrics (2) and (3) corresponding to the energy momentum tensor (4) in bimetric theory can be written explicitly as

$$\left(\frac{B_4}{B}\right)_4 = -16\pi k p \tag{5}$$

$$2\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 = -16\pi k p \tag{6}$$

$$2\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 = 16\pi k \rho \tag{7}$$

where $k = A^2 B$ and hereafter words the suffix 4 after field variable stands for ordinary differentiation with respect to coordinate t.

From Eqs (5) and (6) we obtain

$$\left(\frac{A_4}{A}\right)_4 = \left(\frac{B_4}{B}\right)_4 . \tag{8}$$

By the help of Eq. (8), Eqs (6) and (7) reduce to

$$\rho + 3p = 0 . \tag{9}$$

In view of reality conditions i. e. $p > 0$, $\rho > 0$, the Eq. (9) is true only when $p = 0 = \rho$. Thus in bimetric theory the plane symmetric cosmological perfect fluid model does not survive and hence only vacuum model exists.

For $p = 0 = \rho$ (vacuum case) Eqs (5) and (6) reduce to

$$\left(\frac{B_4}{B}\right)_4 = 0 \tag{10}$$

and

$$\left(\frac{A_4}{A}\right)_4 = 0 . \tag{11}$$

whose solutions can be easily obtained as

$$B = b_1\, e^{a_1 t} \tag{12}$$

and

$$A = b_2\, e^{a_2 t} \tag{13}$$

where a_1, a_2, b_1 and b_2 are constants of integration.

Thus in view of equations (12) and (13), the metric (2) takes the form

$$ds^2 = dt^2 - e^{2a_2 t}(dx^2 + dy^2) - e^{2a_1 t}\, dz^2 \tag{14}$$

which for $a_1 = a_2 = a$ reduces to

$$\mathrm{d}a^2 = \mathrm{d}t^2 - \mathrm{e}^{2st}(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)\,. \tag{15}$$

This vacuum model represents Robertson-Walker flat model, which expands uniformly along space directions with time. The rate of expansion depends on the signature of the parameter s.

4. Massive Scalar Field

In this section we consider the region of the space-time with attractive massive scalar meson field whose energy momentum tensor is given by

$$T_{ij} = V_{,i}V_{,j} - \frac{1}{2}g_{ij}(V_{,m}V^{,m} - M^2V^2) \tag{16}$$

together with

$$g^{ij}V_{;ij} + M^2V = 0 \tag{17}$$

where M is the mass parameter of the scalar meson field V. Hereafterwards the suffix comma and semicolon after a field variable represent ordinary and covariant differentiations with respect to t and g_{ij} respectively.

The explicit form of the field equations (1) for the metrics (2) and (3) with energy momentum tensor (16) are obtained as

$$\left(\frac{B_4}{B}\right)_4 = -8\pi k(V_4^2 - M^2V^2) \tag{18}$$

$$2\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 = -8\pi k(V_4^2 - M^2V^2) \tag{19}$$

$$2\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 = 8\pi k(V_4^2 + M^2V^2) \tag{20}$$

From Eqs (18) and (19) we get

$$\left(\frac{A_4}{A}\right)_4 = \left(\frac{B_4}{B}\right)_4 . \tag{21}$$

which immediately yields

$$B = Aba^t \tag{22}$$

where $a > 0$ and b are constants of integration.

Using Eq. (21) in Eqs (19) and (20), we obtain

$$\left(\frac{A_4}{A}\right)_4 = -8\pi k(V_4^2 - M^2V^2) \tag{23}$$

and

$$3\left(\frac{A_4}{A}\right)_4 = 8\pi k(V_4^2 + M^2V^2) \tag{24}$$

which give

$$2V_4^2 - M^2V^2 = 0\,. \tag{25}$$

This equation yields two basic solutions for V as

$$V = \alpha\, \mathrm{e}^{\frac{M}{\sqrt{2}}t} \tag{26}$$

and

$$V = \beta\, \mathrm{e}^{-\frac{M}{\sqrt{2}}t} \tag{27}$$

where α and β are initial values of V. Klein-Gorden equation (17) for the metric (2) becomes

$$V_{44} + \left(2\frac{A_4}{A} + \frac{B_4}{B}\right) + M^2V = 0\,. \tag{28}$$

Using the values of B and V from the Eqs (22) and (26) in Eq. (28), we get

$$A = ca^{-\frac{t}{3}}\,\mathrm{e}^{-\frac{M}{\sqrt{2}}t}\,. \tag{29a}$$

Using this value of A in Eq. (22) we have

$$B = da^{\frac{2t}{3}}\,\mathrm{e}^{-\frac{M}{\sqrt{2}}t}\,. \tag{29b}$$

where c and d are initial values of A and B respectively.

As before using (22) and (27) in Eq. (28) we get

$$A = Ca^{-\frac{t}{3}}\,\mathrm{e}^{\frac{M}{\sqrt{2}}t}\,. \tag{30a}$$

Subsequently equation (22) yields

$$B = Da^{\frac{2t}{3}}\,\mathrm{e}^{\frac{M}{\sqrt{2}}t} \tag{30b}$$

where C and D are initial values of A and B respectively.

The metrics corresponding to solution (29a, b) and (30a, b) can be written as

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{e}^{-\sqrt{2}Mt}\left[a^{-\frac{2t}{3}}(\mathrm{d}x^2 + \mathrm{d}y^2) + a^{\frac{4t}{3}}\,\mathrm{d}z^2\right] \tag{31}$$

and

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{e}^{\sqrt{2}Mt}\left[a^{-\frac{2t}{3}}(\mathrm{d}x^2 + \mathrm{d}y^2) + a^{\frac{4t}{3}}\,\mathrm{d}z^2\right] \tag{32}$$

respectively.

Here it is sufficient to consider the solution given by (27) and (30a, b) since the behavior of the results are identical for both solutions obtained here except that the model governed by the solution given by Eqs (27) and (30a, b) represents an expanding model whereas the model governed by the solution given by Eqs (26) and (29a, b) represents a contracting model. It can be clearly observed for the expanding model that the scalar field V decreases exponentially with time. The energy density (Anderson [16] p. 289) associated with the scalar field for the expanding model

$$\omega = \frac{1}{2}(V_4^2 + M^2V^2) \tag{33}$$

becomes

$$\omega = \frac{3}{4}\beta^2 M^2 \,\mathrm{e}^{-\sqrt{2}Mt}\,. \tag{34}$$

The spatial volume of the model Vol $= \mathrm{e}^{\frac{3}{\sqrt{2}}Mt}$ indicates that the universe starts evolving with unit volume at initial epoch and at infinite future it blows up for $M > 0$ and annihilates for $M < 0$. For $M = 0$ the scalar field does not survive and both models given by (31) and (32) yield vacuum model as obtained in (14) with $a_1 + 2a_2 = 0$. Thus the models obtained here being dependent on mass of the scalar field are analogous to the mass model obtained by Mohanty and Pradhan [17] in general relativity. Moreover in expanding model (32) the energy density of the scalar field also decreases with time at faster rate than the scalar field V, which is physically acceptable except that throughout evolution $\omega \succ o$ and Vol $\to \infty$, $\omega \to 0$ as $t \to \infty$. Both models (31) and (32) remains anisotropic in nature during evolution.

5. Identification of Scalar Field with Perfect Fluid

The eigenvalues λ_i $(i = 1, 2, 3, 4)$ of the energy momentum tensor T_j^i are given by the determinantal equation

$$|T_j^i - \lambda\delta_j^i| = 0\,. \tag{35}$$

For perfect fluid given by Eq. (4) the eigenvalues are given by

$$\lambda_1 = \lambda_2 = \lambda_3 = -p \qquad \text{and} \qquad \lambda_4 = \rho \tag{36}$$

whereas for massive scalar field given by Eq. (16) are obtained as

$$\lambda_1 = \lambda_2 = \lambda_3 = \frac{1}{2}(M^2V^2 - V_4^2) \quad \text{and} \quad \lambda_4 = \frac{1}{2}(M^2V^2 + V_4^2)\,. \tag{37}$$

Following Tiwari et al [14] if we identify the corresponding eigenvalues of the energy momentum tensors of perfect fluid and scalar meson field from Eqs (36) and (37) we get

$$p = \frac{1}{2}(V_4^2 - M^2V^2) \qquad \text{and} \qquad \rho = \frac{1}{2}(M^2V^2 + V_4^2) \tag{38}$$

which for the vacuum case, i. e. $p = \rho = 0$ studied in Section 3 is true only when

$$V = 0 \qquad \text{or} \qquad V = \text{const} \qquad \text{and} \quad M = 0\,.$$

Both cases lead to vacuum case which is obviously true. Thus the scalar meson field in general does not represent irrotational perfect fluid for the space-time described by metric of the form (2) in bimetric theory. However the parallel result, i. e. the equivalency of these two fields is true in general theory of relativity (Tiwary et al [2] and Tabensky and Taub [3]).

6. Conclusion

In this paper it is shown that in bimetric theory the micro cosmological model representing perfect fluid distribution does not survive whereas the micro cosmological model representing scalar meson field survives when the space-time is described by a metric of the form (2) and $M = 0$. Moreover when $M = 0$ the scalar meson field reduces to zero mass scalar field and in view of Eqs (26) and (27) the scalar field V does not interact with gravitational field. Thus the identification of these two fields done earlier by Tiwari et al [2] and Tabensky and Taub [3] in general theory of relativity is not true in bimetric theory.

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