

**EFFECTS OF ELECTRON - ELECTRON INTERACTIONS AND OF  
MAGNETIC FIELD ON DENSITY OF STATES  
IN DISORDERED METALLIC SYSTEMS**

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**ABSTRACT**

Effect of electron-electron interaction and of magnetic field on the one particle density of states (DOS) is considered for two and three-dimensional weakly disordered metallic systems. High order corrections to the Green's function is taken into account by means of Dyson equation. By summing up infinite series of diagrams in magnetic field for the DOS, it is shown that in two and three dimensions the dependences of the DOS on the magnetic field and energy are nonlinear and it is obtained that at the Fermi level when energy, reckoned from Fermi level, goes to zero the DOS vanishes for both dimensions.

**Density of States in Magnetic Field:**

The energy dependence of the DOS of electronic system can be calculated as  $\nu^{(d)}(\varepsilon) = \sum_{\vec{k}} \delta(\varepsilon(\vec{k}) - \varepsilon)$  or by using Green's function as  $\nu^{(d)}(\varepsilon) = -\frac{2}{\pi} \text{Im} \int \frac{d^{(d)}\vec{k}}{(2\pi)^d} G_R(\varepsilon, \vec{k})$  and the same expression for nonhomogenous systems (e.g. in the presence of an magnetic field) is given by  $\nu^{(d)}(\varepsilon) = -\frac{2}{\pi} \text{Im} G_R(\vec{r}_1, \vec{r}, \varepsilon)$ , where  $G_R$  is total retarded Green's function averaged over impurity concentration. It is defined by  $G_R(\vec{p}, \omega) = [\omega - \varepsilon(\vec{p}) - i/2\tau]^{-1}$ .

As it is known, main contributions to the physical properties of disordered systems in the weak localization theory are connected with two singularities: First appears in the diffusion propagator, characterizing an electron-hole pair with small difference of the momenta  $q$  and of the energies  $\omega$ , (Diffusion pole). Other singularity is due to propagation of electron-electron pairs with small sum of the momenta  $Q$  and small difference of the energies (Cooper pole).

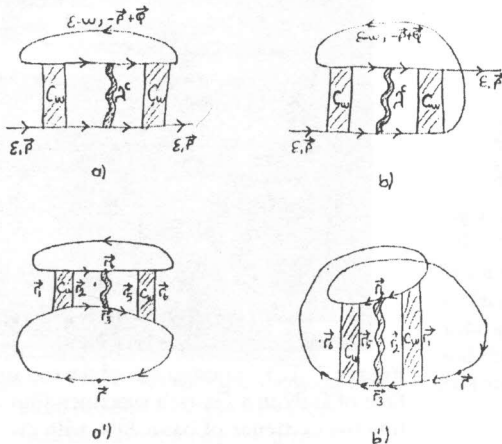


FIGURE 1. a) direct and b) exchange cooperon corrections to the DOS in momentum space; a') and b') the same diagrams in coordinate space.

Altshuler and Aronov calculated the self-energies, showing in Fig. 1, to get a magnetic field dependent quantum correction to the DOS.

We have calculated the DOS in an external magnetic field. High order corrections to the Green's function is taken into account by means of Dyson equation (A. A. Abrikosov, L. P. Gor'kov and I. E. Dzyaloskinskii, 1963)

$$G(\epsilon, \vec{P}) = \frac{1}{G_0^{-1}(\epsilon, \vec{P}) - \sum(\epsilon, \vec{P})} \tag{1}$$

where  $\sum(\epsilon, \vec{P})$  is the sum of all self-energies. In the presence of a magnetic field the Dyson equation written in the real coordinate space is

$$G(\vec{r}, \vec{r}', \epsilon) = G_0(\vec{r}, \vec{r}', \epsilon) + \int G_0(\vec{r}, \vec{r}_1, \epsilon) \sum(\vec{r}_1, \vec{r}_2, \epsilon) G(\vec{r}_2, \vec{r}', \epsilon) \tag{2}$$

In our calculations we have used, particularly, the self-energies drawn in Fig. 1. High order corrections to the Green's function looks as a infinite series shown in Fig. 2.

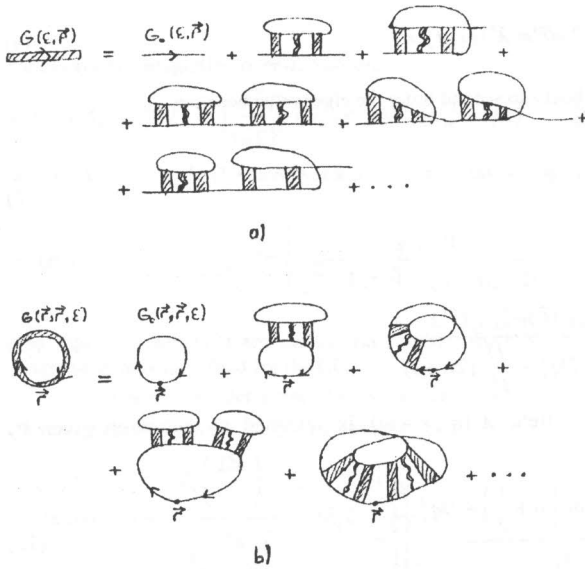


FIGURE 2. Infinite series of high corrections to the Green's function  
a) In momentum space.  
b) In coordinate space.

First of all let's calculate the first correction of Fig. 1a). The expression corresponding to this diagram is

$$\int d\vec{r}_1 \int d\vec{r}_2 \int d\vec{r}_3 \int d\vec{r}_4 \int d\vec{r}_5 \int d\vec{r}_6 G_0^R(\vec{r}, \vec{r}_1, \epsilon) G_0^R(\vec{r}_2, \vec{r}_3, \epsilon) G_0^R(\vec{r}_3, \vec{r}_5, \epsilon) G_0^R(\vec{r}_6, \vec{r}, \epsilon) \times G_0^A(\vec{r}_2, \vec{r}_4, \epsilon - \omega) G_0^A(\vec{r}_4, \vec{r}_5, \epsilon - \omega) G_0^A(\vec{r}_6, \vec{r}_1, \epsilon - \omega) C_\omega(\vec{r}_1, \vec{r}_2, \omega) C_\omega(\vec{r}_5, \vec{r}_6, \omega) \lambda_c(\vec{r}_3, \vec{r}_4) \tag{3}$$

the main problem is reduced to calculation of cooperon  $C_\omega(\vec{r}, \vec{r}', \omega)$  in magnetic field. We define Cooperon as

$$C_\omega(\vec{r}, \vec{r}') = \sum_\eta \lambda_\eta(\omega) \psi_\eta(\vec{r}) \psi_\eta^*(\vec{r}') \tag{4}$$

$$C_0(\vec{r}, \vec{r}') = \sum_\eta \lambda_\eta^0(\omega) \psi_\eta(\vec{r}) \psi_\eta^*(\vec{r}') \tag{5}$$

$$C_\omega(\vec{r}, \vec{r}', \omega) = u^2 \delta(\vec{r} - \vec{r}') + u^2 \sum_{\eta} C_0(\vec{r}, \vec{r}_1, \omega) C_\omega(\vec{r}_1, \vec{r}', \omega) \quad (6)$$

$$\text{where } C_0(\vec{r}, \vec{r}_1) = G_A(\vec{r}, \vec{r}_1, \varepsilon) G_R(\vec{r}, \vec{r}_1, \varepsilon + \omega) \quad (7)$$

Substituting (4) and (5) into Eq. (6) and using orthogonality of  $\{\psi_\eta(\vec{r})\}$ , (i.e.  $\sum_{\eta} \psi_\eta(\vec{r}) \psi_\eta^*(\vec{r}') = \delta(\vec{r} - \vec{r}')$ ) we get  $\lambda_\eta(\omega) = \frac{n_i u^2}{1 - n_i u^2 \lambda_\eta^0(\omega)}$  then  $C_\omega(\vec{r}, \vec{r}', \omega)$  becomes

$$C_\omega(\vec{r}, \vec{r}') = n_i u^2 \sum_{\eta} \frac{\psi_\eta(\vec{r}) \psi_\eta^*(\vec{r}')}{1 - n_i u^2 \lambda_\eta^0(\omega)} \quad (8)$$

After using quasi-classical condition  $G^{R,A}(\vec{r}, \vec{r}', \omega) = \exp\left\{\mp i \frac{e\hbar}{c} \int_{\vec{r}'}^{\vec{r}} \vec{A}(\vec{l}) d\vec{l}\right\} G^{R,A}(\vec{r} - \vec{r}', \omega)$  and (7), we get

$$\int C_0(\vec{r} - \vec{r}', \omega) \exp\left\{\frac{2ie}{\hbar c} \int_{\vec{r}'}^{\vec{r}} \vec{A}(\vec{s}) d\vec{s}\right\} \psi_n(\vec{r}') d\vec{r}' = \lambda_n^0(\omega) \psi_n(\vec{r}) \quad (9)$$

By expanding  $\vec{A}(\vec{s})$  and  $\psi_n(\vec{r}')$  in (9) about  $r$  to second order the eigenvalue becomes

$$\lambda_n^0(\omega) = 1 - 4\tau D \left(\frac{eB}{\hbar c}\right) \left(n + \frac{1}{2}\right) + i\omega\tau + \tau D q_z^2 \quad (10)$$

Substituting (10) into (8) we get

$$C_\omega(\vec{r}, \vec{r}', \omega) = 2\pi\nu_0 \sum_{n,\alpha}^{(l_n/l)^2} \int \frac{dq_z}{2\pi} \frac{\psi_{n,\alpha,k_z}(\vec{r}) \psi_{n,\alpha,k_z}^*(\vec{r}')}{-i\omega + Dq_z^2 + \frac{D}{L_H^2} (2n+1)} \quad (11)$$

In the presence of magnetic field  $\lambda_c(\vec{q}, 2\varepsilon - \omega)$  is screened by cooperon given by expression (11). As a result, at  $T = 0$

$$\lambda_c(\vec{q}, 2\varepsilon - \omega) = \left[ \lambda_0^{-1} - \ln \left( \frac{-i|2\varepsilon - \omega| + \omega_c \left(n + \frac{1}{2}\right) + Dq_z^2}{\varepsilon_0} \right) \right]^{-1} \quad (12)$$

where  $\lambda_0$  is bare interaction constant and  $\varepsilon_0 = \varepsilon_F$  for Coulomb repulsion.

After calculation the cooperon (11) and the screened Coulomb interaction (12) in the presence of magnetic field, other parts of Eq. (3) can be converted into momentum space. The self-energy part differs from (3) only by neglecting  $G_0^2(\vec{p}, \varepsilon)$  in (3). As a result we get for the self-energy part  $\sum(\vec{p}, \varepsilon)$

$$\Sigma(\bar{p}, \varepsilon) = -\frac{2i}{\tau^2} \frac{2eB}{\hbar c} D G_0^A(\bar{p}, \varepsilon) \int_0^\infty \frac{d\omega}{2\pi} \int \frac{dq_z}{2\pi} \sum_{n=0}^{(L_H/L)^2} \lambda_c(2\varepsilon - \omega, \bar{B}, q_z) \frac{\tanh \frac{\omega + \varepsilon}{2T} + \tanh \frac{\omega - \varepsilon}{2T}}{\left[-i\omega + Dq_z^2 + \frac{D}{L_H^2}(2n+1)\right]^2} \quad (13)$$

Then, by using the Dyson equation in momentum space for the DOS we get

$$v_d(\varepsilon, \bar{B}, T) = -\frac{2}{\pi} \text{Im} \int \frac{d^d p}{(2\pi)^d} \sum_{k=0}^\infty \beta^k (G_0^R(\varepsilon, \bar{p}))^{k+1} (G_0^A(\varepsilon, \bar{p}))^k = -\frac{2}{\pi} \text{Im} \sum_{k=0}^\infty \beta^k A_k^{(d)}(\varepsilon) \quad (14)$$

where,  $\beta(\varepsilon, T; \bar{B})$  is the self-energy (13) without  $G_0^A(\bar{p}, \varepsilon)$ ,  $\beta = \frac{i}{\tau^2} \beta_0$  and

$$\beta_0(\varepsilon, T; \bar{B}) = -\frac{2iD}{L_H^2} \int_0^\infty \frac{d\omega}{2\pi} \int \frac{dq_z}{2\pi} \sum_{n=0}^{(L_H/L)^2} \lambda_c(2\varepsilon - \omega, \bar{B}, q_z) \frac{\tanh \frac{\omega + \varepsilon}{2T} + \tanh \frac{\omega - \varepsilon}{2T}}{\left[-i\omega + Dq_z^2 + \frac{D}{L_H^2}(2n+1)\right]^2} \quad (15)$$

for 2D system integration over  $q_z$  vanishes.

$$\text{In Eq. (14)} \quad A_k^{(d)}(\varepsilon) = \int \frac{d^d \bar{p}}{(2\pi)^d} (G_0^R(\varepsilon, \bar{p}))^{k+1} (G_0^A(\varepsilon, \bar{p}))^k = A_0^{(d)} \tau^{2k} 2\pi i \frac{k(2k-1)!}{(k!)^2} \quad (16)$$

where,  $A_0^{(d)} = -v_0^{(d)}$ . The sum over  $k$  in (14), after taking into account Eq. (16), can be taken exactly

$$v_d(\varepsilon, \bar{B}, T) = v_0^{(d)} - 4v_0^{(d)} \text{Im} \left\{ \frac{\beta_0(\varepsilon, T; \bar{B})}{\sqrt{1 - 4i\beta_0} (1 + \sqrt{1 - 4i\beta_0})} \right\} \quad (17)$$

The Eqs. (15) and (17) are the central results, which are obtained by summing of infinite series of diagrams for the DOS.

Let us consider point like interaction when  $\lambda_c$  does not depend on neither  $\omega$  nor  $\{q_z, n\}$ .

Then in three dimensions Eq. (15) gives at  $\varepsilon \gg T$

$$\beta_0(\varepsilon, \bar{B}) = \text{const} \frac{\lambda_c \left(\frac{D\hbar}{\varepsilon}\right)^{5/2}}{\varepsilon \left(\frac{\hbar c}{2eB}\right)^2} = i\lambda_c C_1 \frac{B^2}{\varepsilon^{3/2}} \quad (18)$$

in two dimensions we get at  $\varepsilon \gg T$

$$\beta_0(\varepsilon, \bar{B}) = \text{const} \frac{\lambda_c D\hbar}{\varepsilon^2 L_H^4} = i\lambda_c C_2 \frac{B^2}{\varepsilon^2} \quad (19)$$

where  $C_1$  and  $C_2$  are constants. Substituting into Eq. (17) we obtain in three dimensions

$$\nu_{3d}(\varepsilon, \vec{B}) = \nu_{03} - 4\nu_{03} \frac{\lambda_c C_1 \frac{B^2}{\varepsilon^{3/2}}}{\sqrt{1 + 4\lambda_c C_1 \frac{B^2}{\varepsilon^{3/2}} \left( 1 + \sqrt{1 + 4\lambda_c C_1 \frac{B^2}{\varepsilon^{3/2}}} \right)}} \quad (20)$$

in two dimensions

$$\nu_{2d}(\varepsilon, \vec{B}) = \nu_{02} - 4\nu_{02} \frac{\lambda_c C_2 \frac{B^2}{\varepsilon^2}}{\sqrt{1 + 4\lambda_c C_2 \frac{B^2}{\varepsilon^2} \left( 1 + \sqrt{1 + 4\lambda_c C_2 \frac{B^2}{\varepsilon^2}} \right)}} \quad (21)$$

It can be seen from Eqs. (20) – (21) that the dependences of the DOS on the magnetic field  $B$  and energy are nonlinear ones.

At the Fermi level when  $\varepsilon \rightarrow 0$  the DOS vanishes for both dimensions.

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