Bulgarian Journal of Physics 23 Nos 1/2 (1996) 43–50

PHASE TRANSITIONS AND DISSYMMETRY

M. BUSHEV
G. Nadjakov Institute of Solid State Physics
Bulgarian Academy of Sciences
72 Tzarigradsko Chaussee Blvd., 1784 Sofia, Bulgaria

Received 15 February 1996

Abstract. Phase transitions, equilibrium and nonequilibrium, are hallmarked by symmetry changes. The latter are phenomenologically expressed by the introduction of order parameters. The external influences, however, which are represented, generally have different symmetry. The relationship between the symmetry of external influences and the symmetry of emerging phases is considered on the basis of Curie's principle connecting dissymmetries of causes to those of consequences. The difference is expounded between forced (organized) and spontaneous (self-organized) ordering in various condensed-matter systems.

Causality is a rather rare bird in thermodynamics. This observation seems quite natural as thermodynamics deals mainly with states rather than processes, which is why it is often called thermostatics as well. The topic of cause and sequence is briefly touched upon only by the Le Chatelier–Braun principle stating that the reaction of a system to an external action is such as to minimize its effect [1]. Further elaboration of this principle is carried out in nonequilibrium thermodynamics: the theorems of Prigogine [2] and of Glansdorff–Prigogine [3]. Still there is not any explicit reference to the relationship between the symmetry properties of causes and ensuing sequences.

Phase transitions make no exception to this effect, notwithstanding the marked progress in the understanding of equilibrium, as well as of nonequilibrium, phase transitions. This is a rather peculiar fact, having in mind that the most important steps in this field were achieved through the application of symmetry concepts, such as order parameter, spontaneous symmetry breaking, renormalization group, etc. [1, 4, 5]. An important aspect of this symmetry approach is the phenomenological introduction of symmetry properties and symmetry changes, with no reference to the external factors which actually cause such changes.

The field of synergetics which recently was developed (see, e. g. [6] and references therein) brought to the fore the importance of the notion of control parameter: the physical characteristic of the external influence whose variations bring about structural, i. e. symmetry changes in the system.

A simple example of structuring under an external influence is the magnetic ordering of a paramagnet in a magnetic field **H**, which induces a macroscopic magnetization **M**. The original rotational symmetry of the paramagnet is broken by the vector **M**, which is induced by the control parameter **H**. This is an obvious example of an induced symmetry.

Landau's theory of second-order phase transitions describes, on the other hand, spontaneous symmetry breaking. Before going any further, we should immediately point out that spontaneous is by no means causeless. Take again the simple example of a paramagnet. When cooled below Curie's temperature T_C, it undergoes a phase transition to a ferromagnetic state with a spontaneously broken symmetry. Here, apparently, the control parameter, or the cause, is the temperature difference $(T - T_C)$. Depending on the sign of $(T - T_C)$, the order parameter can be either vanishing, and then we have a disordered phase of higher symmetry, or nonvanishing, when the phase is ordered and the symmetry is broken spontaneously.

Here, obviously, it is not the lack of cause, but the specific relationship between the symmetry of the cause and the symmetry of the resulting state which determines the spontaneity of the process.

The advent of synergetics, whose main object of study is the self-organization of systems, urged upon the importance of a rigorous distinction between processes of organization and self-organization. Clearly, both types of processes take place in open systems. Synergetics studies nonequilibrium systems in which flows of energy and substance bring about specific types of ordering, or spontaneous symmetry breaking.

There are at least three good reasons why a rigorous definition of organization and self-organization should be sought for that would establish a clear-cut distinction between these two notions.

First, we remind that cybernetics was perhaps the first field of study which realized the importance of the concept of self-organization. Many authors, among them W. Ashby, N. Wiener and J. Neumann, dwelled upon this notion, and in the early sixties cyberneticians devoted many discussions to the principles of self-organization. However, in the editorial to the proceedings of a symposium on this topic we read: "In the course of many ages self-organization has remained perhaps the most puzzling phenomenon, the most cherished secret of nature" [7]. The sharpness of this problem was realized along with the thorough development of the theory of feedback, inverse and positive, which came close to discovering the crucial role of nonlinearities in the origin of structures.

Second, it was synergetics which in the seventies picked up the problem in a still broader scope. This field of study had a quick start after the realization (R. Graham and H. Haken, 1968) of the close analogy between laser radiation and superconducting phase transitions. This opened the way for a dramatic progress in the understanding of nonequilibrium phase transitions. One could discern three main sources of such transitions, namely, far-from-equilibrium processes, nonlinear effects and cooperative phenomena. These sources became starting points, correspondingly, of the thermodynamic theory of dissipative structures (the Brussels school of I. Prigogine), of the theory of dynamic systems (the dynamic models of Lotka and Volterra, of Verhulst, of

A. Turing, of E. Lorenz, and many others), and of the mesoscopic approach based on the Fokker-Planck equation (H. Hakken and his school) [8]. However, in spite of the wealth of achievements, synergetics, whose main subject matter is self-organization, operates largely with an intuitive understanding of this notion. This was plainly stated in a recent paper [9] on the statistical theory of open systems and the criteria of the degree of order in the processes of self-organization. In spite of the fact, wrote Yu. Klimontovitch, that presently ample literature is devoted to the theory of self-organization, one cannot find in it an unambiguous answer to the question: What is self-organization? It is the understanding of this author, he continued, that instead of looking for the answer of this question, it would be better to find out the criteria of the degree of order, which would allow to establish "the very presence of a process of self-organization".

But then the questions arise: how could we be confident that an order of some kind is exactly self- organization, and what is the use of a criterion of something which is not by itself well defined?

The third reason why we need a clear-cut understanding of self-organization lies in a specific situation existing in quantum field theory, namely, the theory of spontaneously broken global and gauge symmetries (see, e. g., L. Ryder [10]). According to Goldstone's theorem (J. Goldstone, 1961), the spontaneous breaking of a continuous global symmetry results in the birth of massless particles — Goldstone bosons (modes). Massive particles appear at the spontaneous breaking of a gauge (local) symmetry — the Higg's-mechanism (P. Higgs, 1964). A symmetry is spontaneously broken when the Hamiltonian of a system possesses certain symmetry, while the stable physical states, including the vacuum state, described by it, lack such symmetry. The symmetric states are unstable and under the action of infinitesimal perturbations pass spontaneously into asymmetric states. This leads to a degenerate ground (stable) state.

What is the physical meaning of this spontaneous breaking of symmetry? The formal presentation of Goldstone's theorem and the Higgs' mechanism creates the impression that this is a causeless process. Indeed, following Ryder (Ch. 8.3. in [10]), we write down the gauge invariant Lagrangian for the static case:

$$-L = \frac{1}{4}(\nabla \times \mathbf{A})^2 + \frac{1}{2}|(\nabla - ie\mathbf{A})\varphi|^2 + m|\varphi|^2 + \lambda\varphi^4 \quad (1)$$

(all notations as in [10]). By formally putting the parameter $m^2 < 0$, it is arrived at the conclusion that the ground (vacuum) state corresponds to a nonvanishing value of the field:

$$|\varphi| = a \equiv \sqrt{\frac{-m^2}{2\lambda}}\,. \quad (2)$$

After some formal transformations it is shown that the vacuum is degenerate, the symmetry is spontaneously broken and the Lagrangian (1) describes two massive fields: photons of spin 1 and Higgs' particles of spin 0. Their respective masses are proportional to a

$$\mu_{\mathrm{ph}} \sim ea, \qquad \mu_{\mathrm{H}} \sim \sqrt{\lambda}a\,. \quad (3)$$

It is the formal change of the sign of m^2 which brings to spontaneous symmetry breaking and by this token gives birth to massive, or massless, bosons. No mention is made of what causes this symmetry breaking, or what causes the change of sign of m^2.

But now we observe that $-L$ in (1) is the Ginzburg–Landau functional describing superconducting phase transitions with

$$m^2 = \alpha(T - T_c), \qquad \alpha > 0\,; \tag{4}$$

T_c being the critical temperature and φ, Eq. (1), — the many-body wave function in the Bardeen–Cooper–Schrieffer theory of superconductivity, or Landau's phenomenological order parameter. Quite obviously, here the temperature is the control parameter and for $T < T_c$ we have Eq. (2) demonstrating spontaneous symmetry breaking. Then the inverse masses in (3) correspond to the two length-scales in type II superconductors, namely, the penetration depth of the magnetic field and the correlation length of vortices. Clearly, it is the control parameter $m^2 < 0$, i. e. the temperature, which causes the spontaneous symmetry breaking with the sequences ensuing.

The impression that quantum field theory puts the cart before the horse is partially rectified in the theory of quasi-averages (N. Bogoliubov, 1961), as well as in the Goldstone–Salam–Weinberg (1962) effective potential approach to spontaneous symmetry breaking. In both methods the degeneracy of the vacuum state is removed through the introduction of an infinitesimal source field. This allows for a physically correct definition of the symmetry breaking parameter m^2 (cf. Coleman [11], Ch. 3.3).

Here immediately the question arises: why the symmetry breaking is spontaneous if it is induced by an external field source? Well, the answer to this question brings us actually to the definition of self-organization [12]. Self-organization is a spontaneous process not because it is causeless, but because the symmetry of the consequence is not uniquely determined by the symmetry of the cause, the control parameter. And vice versa, when the symmetry of the ordered state is uniquely forced by the symmetry of the control parameter, then the system undergoes induced symmetry breaking, or organization.

The terms "organization" and "self-organization" are not met in the dictionary of physics. In physics one speaks normally of induced as well as of spontaneous symmetry breaking. Both couples of terms mean essentially the same.

Table 1. Phase transitions, order parameters, conjugate quantities (fields) and broken symmetry

Phase transition	Order parameter	Conjugate field	Broken symmetry
Liquid–gas	change of density $\rho_L - \rho_G$	chemical potential μ	unspecified
Paramagnet–ferromagnet	magnetization $\mathbf{M}$	magnetic field $\mathbf{H}$	rotational
Paramagnet–antiferromagnet	sublattice magnetization $\mathbf{M}_i$	none	rotational
Paraelectric–ferroelectric	polarization $\mathbf{P}$	electric field $\mathbf{E}$	rotational
Normal metal–superconductor	energetic gap Δ	none	gauge
Normal liquid–superfluid	condensate wave function Ψ_0	none	gauge

The above examples of a paramagnet undergoing magnetic ordering under the effect either of an external magnetic field or of decreased below T_c temperature present illustrations of the two distinct types of symmetry breaking. In Table 1 some equilibrium phase transitions are adduced, the order parameters, the relevant broken symmetries and the conjugate fields which induce the same type of broken symmetry as the spontaneous one. A curious and still not quite understood fact leaps to the eye: in some cases there are no physical fields conjugate to the order parameter. For some deeply hidden reasons not all ordering processes in nature have a corresponding "organizer".

In Table 2 only a few of the multitudinous nonequilibrium phase transitions are shown. A crucial fact about them is that they all represent spontaneous symmetry breaking: there is no conjugate, organizing field. This explains why the field of study of nonequilibrium phase transitions, namely synergetics, is said to have self-organization as a subject matter.

In order to make explicit the relation between the symmetry of causes and the symmetry of consequences, we shall refer to the principle of Pierre Curie (1894), as this principle represents the symmetry aspects of causality. According to one formulation of this principle, a system, subject to external influence, changes its symmetry in such a way that only those elements remain which are common with those of the external influence [13].

Another formulation of the principle makes use of the notion of dissymmetry, introduced by L. Pasteur (1848) as a denotement of a "spoiled symmetry". The dissymmetry of an object x is the set theoretical complement of its symmetry group G_x to some universal group $\tilde{G}$. In other words, the dissymmetry D_x is the cross-section:

$$D_x = \tilde{G} \cap G_x = \tilde{G}/G_x \,, \tag{5}$$

As symmetry reflects the static (invariant) properties of object (processes), so dissymmetry features their dynamics [15], and broken symmetry, in particular, means increased dissymmetry.

Table 2. Some nonequilibrium phase transitions, the corresponding structures, and the relevant control parameters

Nonequilibrium phase transition	Emerging structures	Control parameter(s)
Rayleigh–Bénard instability	rollers, Bénard cells, etc.	Rayleigh number, Prandtl number
Taylor effect	Taylor vortices	Taylor number
Laser light	coherent light	inverse population
Nonstationary Josephson effect	alternating current	permanent applied voltage
Gunn effect	current oscillations	permanent applied voltage
autovibrations	undamped oscillations	relation of constant energy input to dissipated energy
solitons	solitary waves of conserved shape and velocity	relation of constant energy input to dispersed energy

According to Curie's principle, the resulting dissymmetry D_S^R of a system equals the conjunction of the initial dissymmetry of the system D_S^0 and the dissymmetry of the cause D_C, namely:

$$D_S^R = D_S^0 \cup D_C \,. \tag{6}$$

Now we split the dissymmetry of the cause into an external part D_C^e and an internal part D_C^I (which could be called hidden dissymmetry — an obvious allusion to hidden symmetry; [11]):

$$D_C = D_C^e + D_C^i \,. \tag{7}$$

The idea behind this splitting is the following. The explicit form of D_C^e is presumably known in every specific case. For example, in the case of a ferromagnetic transition at $T \leq T_c$ we have $D_C^e = D_T = 0$, as the temperature is a scalar field. Then the hidden dissymmetry D_C^i would correspond to the molecular magnetic field, proportional to the nonvanishing order parameter M (Eq. (4)). Generally speaking, D_C^i is the dissymmetry of the mean field, corresponding to the nonvanishing order parameter.

Quite obviously, $D_C^i \neq 0$ corresponds to spontaneous symmetry breaking, or self-organization. On the other hand, when $D_C^i = 0$, but $D_C^e \neq 0$, we have a case of induced (forced) symmetry breaking, i. e. organization. Thus we arrive at the following definitions of:

(i) Induced symmetry breaking, or organization:

$$D_S^R = D_S^0 \cup D_C^e \qquad \text{with} \qquad D_C^e \neq 0\,; \tag{8}$$

(ii) Spontaneous symmetry breaking, or self-organization:

$$D_S^R = D_S^0 \cup D_C^e \cup D_C^i \qquad \text{with} \qquad D_C^i \neq 0\,. \tag{9}$$

Let us take a few simple examples to illustrate these results. We consider a crystal with initial electron distribution $\rho(\mathbf{R})$, $\mathbf{R}$ — radius vector. Cooling it below certain critical temperature T_c gives birth to a nonvanishing order parameter — the local variation of the electron density $\delta\rho(\mathbf{R})$. Thus the electron density distribution function becomes $\rho(\mathbf{R}) = \rho_0(\mathbf{R}) + \delta\rho(\mathbf{R})$. Following Eq. (7), we have: $D_C = D_T \cup D_{\delta\rho}$. As the dissymmetry $D_T = 0$, we come to the relation between dissymmetries:

$$D_\rho^R = D_\rho^0 \cup D_{\delta\rho} \,. \tag{10}$$

which shows that the dissymmetry of the ordered phase is higher than that of the initial state. This is a spontaneous symmetry breaking, or self-organization.

A simple illustration of the distinction between induced and spontaneous symmetry breaking offers the case of a paramagnet. By applying an external magnetic field H, we come to induced symmetry breaking (organization), described by the equation:

$$D_S^{\mathrm{M}} = D_S^0 \cup D_{\mathrm{H}} \,. \tag{11}$$

The case of spontaneous magnetization, i. e. of self-organization, is described by the equation:

$$D_S^{\mathrm{M}} = D_S^0 \cup D_T \cup D_C^i \,. \tag{12}$$

in which D_S^0 is the dissymmetry of the paramagnetic state, $D_T = 0$ and $D_C^i \neq 0$ is the dissymmetry related to the molecular field. When $D_C^i = 0$, we have the trivial case of temperature variation in the paramagnetic range $T > T_C$.

Consider now the case of piezoelectricity: electrical polarization of crystals submitted to mechanical deformations. Pierre and Jacques Curie predicted this phenomenon (1880) basing on the principle of Curie. Now we understand that this is a case of induced symmetry breaking, i. e. organization: mechanical distortions cause uniquely determined electric polarization, and vice versa. As the tensor character of both deformation and polarization is known, it follows from Eq. (8) that only crystals of specific symmetry classes, complying with (8), would be piezoelectrics (P. and J. Curie predicted the effect for quartz and a few other crystals). Essentially the same refers to piezomagnetizm: the reversible relation between magnetostriction and magnetization under deformation determines, in compliance with Eq. (8), which crystals are piezomagnetic.

Now we can qualify the role of the cause(s) in the processes of organization and self-organization. From what was said above we understand that the cause either induces broken symmetry (organization), or brings the system to the verge of criticality (point of bifurcation), in which any contingency which makes impossible any deterministic description of the transition to ordered state. This explains why I. Prigogine calls such processes "order through fluctuations". It is this combination of causal description with random, unpredictable, behavior which also leads to deterministic chaos (Y. Sinai, 1962; E. Lorenz, 1963) and strange attractors (D. Ruelle, F. Takens, 1971). Both ordering and deterministic chaos have essentially one and the same causal background, depending on the intensity of the control parameter(s).

Thus we come to the conclusion that the numerous spontaneous processes, in quantum theory and elsewhere, though being contingent, with certain probability, still are causally determined by external effects. Such an insight might appear contradictory to the widespread view that spontaneous processes, such as decay of elementary particles or of nuclei, are independent of external influences. This, however, seems not to be a serious stumbling block in view of the theoretically possible quantum Zeno effect [16, 17], which shows that the decay of a system never takes place if it is under a continuous observation ("a watched pot never boils"). Physically, the effect is based on broken time-symmetry of decay processes caused by repeated perturbations. This effect is still to be experimentally verified, but its theoretical plausibility even at present supports the view of the causal nature of spontaneous processes.

References

1. L. D. Landau and E. M. Lifschitz. *Statistical Physics* (Reading 1968).
2. I. Prigogine. In: *Introduction to Nonequilibrium Thermodynamics* (Wiley, New York 1962).
3. P. Glansdorff, I. Prigogine. *Thermodynamic Theory of Structure, Stability and Fluctuations* (Wiley, New York 1971).

4. S. K. Ma. *Modern Theory of Critical Phenomena* (Benjamin 1976).
5. D. Forster. *Hydrodynamic Fluctuations, Broken Symmetry and Correlation Functions* (Benjamin, London 1975).
6. H. Haken. *Rep. Progr. Phys.* **52** (1989) 515.
7. The Principles of Self-Organization — Transactions of the University of Illinois, Symposium on Self-Organization, June 1961 (Preface by A. Lerner to the Russian translation, Mir, Moscow 1966).
8. M. Bushev. *Synergetics: Chaos, Order, Self-Organization* (World Scientific, 1994).
9. Yu. L. Klimontovitch. *Uspekhi Fiz. Nauk* **158** (1989) 59.
10. L. Ryder. *Quantum Field Theory* (Cambridge Univ. Press, 1985).
11. S. Coleman, *Hidden Symmetry: An Introduction to the Theory of Spontaneous Symmetry Breaking and Gauge Fields*, in Laws of Hadronic Matter (Acad. Press, 1975).
12. M. Bushev. *Self-Organization and Dissymmetry*, in Methodology of Mathematical Modelling (Bulg. Acad. Sci, 1992), p. 42.
13. Modern Crystallography (Nauka, Moscow 1981) (in Russian).
14. H. Weyl. *Symmetry* (Princeton Univ. Press, 1952).
15. A. Shubnikov and V. Koptsik. *Symmetry in Science and Art* (New York 1974).
16. L. Khalfin. *Uspekhi Fiz. Nauk* **160** (1990) 185.
17. T. Petrosky, S. Tasaki, I. Prigogine. *Physica* **A170** (1991) 306.