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SCALAR-ELECTROMAGNETIC INTERACTION AND THE CONFINING AND SCREENING PROPERTIES OF ELECTROMAGNETISM

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Abstract. By studying a generic Born–Infeld lagrangian wherein the dimensional constant in the theory is replaced by a pseudoscalar field we study the effect that the pseudoscalar field has on the E. M. field of a point charge. We also study the screening effects that such a pseudoscalar E. M. theory generates for an isolated dyon surrounded by a pseudoscalar domain wall.

1. Introduction

The last six years of heavy ion research has clearly demonstrated that not all is understood in the theory of quantum electrodynamics which once was believed to be an exact and calculable theory in perturbation theory [1, 2, 3]. The investigation of possible 1.8 MeV peaks in heavy ion collisions has opened up a new avenue of research in both QED and cosmology since any anomalous electromagnetic effects will leave their signature in the spectra of gamma ray bursts and extragalactic radiation now being observed [4, 5]. There have been numerous explanations of the anomalous (1.8 MeV e^+e^-) peaks produced in heavy ion collisions which include the creation of a scalar [6], the intermediate state of colored leptons, [7] and the possible production of a quasi-super heavy atom whose surface vibration mode oscillates with the 1.8 MeV value [8]. However, the most plausible and acceptable theory is that the elecromagnetic field suffers an altered vacuum structure in whose background an e^+e^- pair has a total energy of 1.8 MeV [9]. This vacuum state is confining and thus the perturbative structure of QED breaks down in this energy range. If such a modified vacuum state of QED appears, it is natural to ask what other particles might be produced in it and how might we represent the state in a semi-classical way independent of the operator formalism of Quantum Field Theory. The subject of the present paper is to study the modifications brought about in electrodynamics by the presence of a scalar or pseudoscalar field coupled to electromagnetism. We choose to study a Born-Infeld type [10] lagrangian wherein the dimensional constant is replaced by a pseudoscalar field which effectively generates a spatially varying self-coupling strength. By studying the equation of the EM field and pseudoscalar field we may ascertain the confining properties of such a coupled field system. Also by introducing a domain wall [11, 12] outside a bare dyon charge we may study how a domain wall screens, anti-screens and, in general, alters the observable charge to a distant observer.

2. Electromagnetism Coupled to a Pseudoscalar Field

We begin our analysis by writing the following Born-Infeld matter lagrangian of the electromagnetic field coupled to a pseudoscalar field wherein the Born-Infeld [13] constant is replaced by

$$b \to \frac{\phi^2}{l_0^2}$$

ϕ = pseudoscalar field, l_0 = fundamental lenght

$$\mathcal{L} = -\frac{\phi^2}{8\pi l_0^2}\left(\sqrt{1+\frac{Jl_0^2}{\phi^2}-\frac{I^2 l_0^4}{16\phi^4}}-1\right)\sqrt{-g} + \frac{\partial_\mu\phi\partial^\mu\phi}{2}\sqrt{-g} - \frac{A_2}{4}\left(\phi^2-\frac{A_1}{A_2}\right)^2\sqrt{-g} \tag{2.1}$$

here

$$J = F_{\mu\nu}F^{\mu\nu}\,,$$

$$I = \frac{\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}F_{\mu\nu}}{\sqrt{-g}}\,.$$

For the metric we have

$$(dS)^2 = e^{\nu}(dx^4)^2 - e^{\lambda}(dr)^2 - r^2(d\theta)^2 - r^2\sin^2\theta(d\phi)^2\,, \tag{2.2}$$

the total lagrangian of gravity plus matter is

$$\mathcal{L} = \frac{c^4}{16\pi G}R\sqrt{-g} + \mathcal{L}_M\,. \tag{2.3}$$

Varying Eq. (2.1) with respect to A_μ gives

$$\frac{\partial}{\partial x^\nu}\left(\frac{\frac{1}{4\pi}F^{\mu\nu}\sqrt{-g}}{\sqrt{1+\frac{Jl_0^2}{\phi^2}-\frac{I^2 l_0^2}{16\phi^4}}}\right) - \frac{\partial}{\partial x^\nu}\left(\frac{l_0^2(\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta})I}{32\pi\phi^2\sqrt{1+\frac{Jl_0^2}{\phi^2}-\frac{I^2 l_0^4}{16\phi^4}}}\right) = 0\,. \tag{2.4}$$

Varying with respect to ϕ gives

$$-\Box\phi - A_2\phi\left(\phi^2 - \frac{A_1}{A_2}\right) - \frac{\phi}{4\pi l_0^2}\left(\sqrt{1 + \frac{Jl_0^2}{\phi^2} - \frac{I^2 l_0^4}{16\phi^4}} - 1\right)$$
$$-\frac{\phi^2}{16\pi l_0^2}\left(\frac{-\frac{2Jl_0^2}{\phi^3} + \frac{I^2 l_0^4}{4\phi^5}}{\sqrt{1 + \frac{Jl_0^2}{\phi^2} - \frac{I^2 l_0^4}{16\phi^4}} - 1}\right) = 0\,. \tag{2.5}$$

If in the above equations we neglect the terms containing I we have upon integration of Eq. (2.4) (assuming flat space $e^\nu = e^\lambda = 1$)

$$\frac{r^2 E}{4\pi\sqrt{1 - \frac{2E^2 l_0^2}{\phi^2}}} = \frac{e}{4\pi} \tag{2.6}$$

($e =$ central electric charge), and Eq. (2.5) becomes

$$\frac{1}{r^2}\frac{d}{dr}(r^2\phi_{,r}) = A_2\phi\left(\phi^2 - \frac{A_1}{A_2}\right)$$
$$+\frac{\phi}{4\pi l_0^2}\left(\sqrt{1 - \frac{2E^2 l_0^2}{\phi^2}} - 1\right) - \frac{\frac{E^2}{4\pi\phi}}{\sqrt{1 - \frac{2E^2 l_0^2}{\phi^2}}}\,. \tag{2.7}$$

Substituting Eq. (2.6) into Eq. (2.7) gives

$$\frac{d}{r^2 dr}(r^2\phi_{,r}) = A_2\phi\left(\phi^2 - \frac{A_1}{A_2}\right)$$
$$+\frac{\phi}{4\pi l_0^2}\left(\sqrt{\frac{r^4}{r^4 + \frac{2e^2 l_0^2}{\phi^2}}} - 1\right) - \frac{e^2}{4\pi\phi r^2\sqrt{r^4 + \frac{2e^2 l_0^2}{\phi^2}}}\,. \tag{2.8}$$

To solve Eq. (2.8) we have to resort to either numerical methods or a power series approach. Let us call

$$\phi_{R_1},\ \left(\frac{d\phi}{dr}\right)_{R_1}$$

the values of the pseudoscalar field and its derivative at $r = R_1$ (here R_1 — small radius surrounding charge e), then from Eq. (2.8) we have

$$\phi_{,rr} = \frac{1}{r^2}\left[\begin{array}{c} r^2 A_2 \phi\left(\phi^2 - \frac{A_1}{A_2}\right) + \frac{r^2\phi}{4\pi l_0^2}\left(\sqrt{\frac{r^4}{r^4 + \frac{2e^2 l_0^2}{\phi^2}}} - 1\right) \\ -2r\phi_{,r} - \frac{e^2}{4\pi\phi\sqrt{r^4 + \frac{2e^2 l_0^2}{\phi^2}}} \end{array}\right]. \tag{2.9}$$

If the potential term in Eq. (2.9) dominates and if

$$(\phi)_{r=R_1} < \sqrt{\frac{A_1}{A_2}}, \quad (\phi_{,rr})_{R_1} < 0\,,$$

then

$$\phi(r) = \phi_{R_1} + \left(\frac{d\phi}{dr}\right)_{R_1}(r - R_1) + \frac{1}{2}\left(\frac{d^2\phi}{dr^2}\right)_{R_1}(r - R_1)^2 \tag{2.10}$$

and the quadratic term will give a decreasing value of $\phi(r)$ away from $r = R_1$. From the solution of Eq. (2.6) we have

$$E = \frac{e}{\sqrt{r^4 + \frac{2e^2 l_0^2}{\phi^2}}}\,. \tag{2.11}$$

If ϕ increases away from the point $r = R_1$ then the bare charge will be anti-screened from Eq. (2.11), however if ϕ decreases away from $r = R_1$ the bare charge e will be screened from Eq. (2.11). In early universe studies if alternating regions of false vacuum ($\phi = 0$) and true vacuum

$$\left(\phi = \sqrt{\frac{A_1}{A_2}}\right)$$

are present then most likely particles are created near the region of true vacuum (where the Higgs-field gives up its energy to particle creation) [14], thus away from the particle's core in a classical sense, the pseudoscalar field will decrease (more toward the false vacuum) and bare electric charge would be screened from Eq. (2.11). If the potential term in Eq. (2.8) does not dominate them a numerical solution of Eq. (2.8) would have to be obtained to uncover the charge screening or anti-screening properties of the coupled system in Eqs (2.6) and (2.7). We also note that if ϕ decreases away from the core of the practicle then the screening properties of the pseudoscalar field would lead to a notion of deconfinement since the effective electric field would decrease relative to the value it would have when $\phi =$ constant.

3. Screening of Electric Charge in a Pseudoscalar-Electromagnetic System in the Presense of a Domain Wall

To discuss the screening effects of an electric charge by a pseudoscalar domain wall we add a term to the lagrangian in the Eq. (2.1) motivated by axion-like models [15].

$$\begin{aligned}\mathcal{L}_M = &-\frac{\phi^2}{8\pi l_0^2}\left(\sqrt{1+\frac{Jl_0^2}{\phi^2}}-1\right)\sqrt{-g}+\frac{\partial_\mu\phi\partial^\mu\phi}{2}\sqrt{-g} \\ &-\frac{A_2}{4}\left(\phi^2-\frac{A_1}{A_2}\right)^2\sqrt{-g}+\alpha\left(\frac{e^{\mu\nu\alpha\beta}F_{\alpha\beta}F_{\mu\nu}}{\sqrt{-g}}\right)\phi\sqrt{-g}\end{aligned} \tag{3.1}$$

(α = coupling constant). Here we have neglected the I^2 terms again in Eq. (2.1), Eq. (2.4) now becomes

$$\frac{\partial}{\partial x^\nu}\left(\frac{F^{\mu\nu}\sqrt{-g}}{4\pi\sqrt{1+\frac{Jl_0^2}{\phi^2}}}\right)-\frac{\partial}{\partial x^\nu}(4\alpha\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}\phi)=0\,. \tag{3.2}$$

Eq. (2.5) becomes

$$\begin{aligned}-\Box\phi-A_2\phi\left(\phi^2-\frac{A_1}{A_2}\right)-\frac{\phi}{4\pi l_0^2}\left(\sqrt{1+\frac{Jl_0^2}{\phi^2}}-1\right) \\ +\frac{\alpha\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}F_{\mu\nu}}{\sqrt{-g}}+\frac{J}{8\pi\phi\sqrt{1+\frac{Jl_0^2}{\phi^2}}}=0\,.\end{aligned} \tag{3.3}$$

We now consider a central dyonic charge e_0, q sitting in the true vacuum state

$$\phi_1=\sqrt{\frac{A_1}{A_2}}\,,$$

since ϕ is constant we have from Eq. (3.2)

$$\frac{\partial}{\partial x^\nu}\left(\frac{\sqrt{-g}F^{\mu\nu}}{4\pi\sqrt{1+\frac{Jl_0^2}{\phi_1^2}}}\right)=0\,. \tag{3.4}$$

Also

$$\frac{\partial}{\partial x^\nu}(\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta})=0 \tag{3.5}$$

from the existence of a potentioal for all r.

Eqs (3.4) and (3.5) give for $r < R_1$,

$$B = \frac{q}{r^2}$$

$$E = \frac{e_0}{\sqrt{r^4 + \frac{2e_0^2 l_0^2}{\phi_1^2}}} . \tag{3.6}$$

Here we assume that the variation of the pseudoscalar field can be neglected for $R_D < r \leq R_1$ (where R_D = effective radius of dyon), also R_1 = inner radius of domain wall. For $r \geq R_2$ we have

$$\left(\phi_2 = -\sqrt{\frac{A_1}{A_2}}\right)$$

$$B = \frac{q}{r^2}$$

$$E = \frac{e}{\sqrt{r^4 + \frac{2e^2 l_0^2}{\phi_2^2}}} . \tag{3.7}$$

Here e = electric charge seen for $r \geq R_2$, we also neglect the variation of ϕ for $r > R_2$. Integrating Eq. (3.2) between R_1 and R_2 gives where we use E_1 and E_2 at R_1 and R_2 from Eqs (3.6) and (3.7)

$$\frac{R_2^2 E_2}{4\pi\sqrt{1 - \frac{2E_2^2 l_0^2}{\phi_2^2}}} - \frac{R_1^2 E_1}{4\pi\sqrt{1 - \frac{2E_1^2 l_0^2}{\phi_1^2}}} = 8\alpha q\left(-\sqrt{\frac{A_1}{A_2}} - \sqrt{\frac{A_1}{A_2}}\right)$$

or

$$e = e_0 - 64\pi\alpha q\sqrt{\frac{A_1}{A_2}} . \tag{3.8}$$

If in the above analysis

$$\phi_{R_1} = -\sqrt{\frac{A_1}{A_2}} , \quad \phi_{R_2} = \sqrt{\frac{A_1}{A_2}}$$

then the relation in Eq. (3.8) would read

$$e = e_0 + 64\pi\alpha q\sqrt{\frac{A_1}{A_2}} .$$

or the bare charge e_0 of the dyon would be anti-screened. We see from Eq. (3.8) that the screening or anti-screening properties of the dyon depend on the condition

$$\frac{\partial}{\partial x^{\nu}}(\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta})=0$$

for the potential and the form of the pseudoscalar E & M coupling in Eq. (3.1). If higher powers of

$$\frac{\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}F_{\mu\nu}}{\sqrt{-g}}$$

appear in the lagrangian, or higher powers of the pseudoscalar field appear in the coupling then the screening relation (Eq. (3.8)) would be altered. In two previous notes [11, 12] we have discussed such screening effects for domain type walls, with implications ranging from the altering of plasma effects to the modification of particle phenomena such as proton decay and reaction rates of elementary particle interactions sensitive to external electric fields.

4. Conclusion

In the above analysis, we have studied the screening and anti-screening properties of a coupled electromagnetic pseudoscalar theory wherein the dimensional constant in the Born-Infeld theory is allowed to vary as a pseudoscalar field to the quadratic power. Pseudoscalar fields appear in physics for a variety of reasons as do scalar fields. In supergravity theory the scalar dilaton occurs in a low energy effective theory as does the two-index field. It would be of interest to see what effect the dilaton has on electromagnetic interactions of the Born-Infeld type since the Born-Infeld lagrangian has been shown to be low energy effective lagrangian generated from string theory in the field theory limit [16]. It would also be of practical interest to ask what effect pseudoscalar and scalar fields would have on QCD confinement and asymptotic freedom since it is usually accepted that the asymptotic freedom and confinement of QCD depends only on the fermion fields and gauge fields of QCD [17]. If proton decay becomes a subject of discussion then the screening of the monopole field which catalyzes proton decay [18] by the Higgs field would certainly affect both proton decay and reactions leading to baryogenisis. Though the above treatment is semi-classical, it is still suggestive of the fact that both screening by a local pseudoscalar field and by a domain wall are important problems that may play a vital role in early universe studies effecting both baryogenisis and the generation of perturbations that eventually seed large scale structure [19].

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