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VARIATIONAL METHOD FOR THE ANALYSIS OF THREE-LAYER MICROSTRIP-LIKE TRANSMISSION LINES FOR NONDESTRUCTIVE PERMITTIVITY MEASUREMENT

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Abstract. The method presented uses series of expansion of the unknown charge distribution over the strip conductor in terms of Chebychev's polynomials. It takes into account the charge density behaviour near the strip edges. Applications to sensitivity analysis for nondestructive permittivity measurement are discussed. Two new structures are suggested.

Резюме. Представленный метод использует линейную комбинацию полиномов Чебишева для аппроксимации плотности заряда на микрополоске. Учитывается поведение плотности заряда возле краев полоска. Обсуждаются применения к неразрушающим измерениям диэлектрической проницаемости. Предложены две новые структуры.

1. Introduction

Nondestructive permittivity measurement based on microstrip-like transmission lines with one and two dielectric sheets has been presented in [1]. In the present paper we suggest that a three-layer structures should be used for this purpose. The three-layer structures can increase the measurement method's sensitivity in some cases. An analysis method for such structures can be useful to assess some air gap effects by the two-layer microstrip-like transmission lines used for nondestructive permittivity measurement.

Figure 1 shows all three-layer structures that can be made using only one strip conductor.

The structure from Fig. 1*b* can be applied for nondestructive permittivity measurement of dielectric sheets, including metallized ones. The sheet with unknown permittivity can be put between the metal grounded plate and sheet *3* with the microstrip conductor over it both in positions *1* and *2* (Fig. 2). The unknown permittivity can be computed from the measured guided wavelength in the structure and the known geometry and permittivities of the other two sheets (one of them being air).

Another possible application of the three-layer structure analysis method is the investigation of the air gaps by two layer structures. Such air gaps arise when the

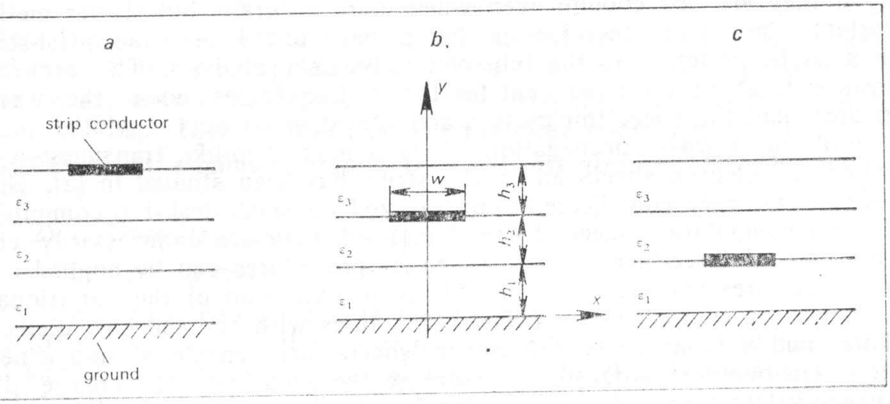

Fig. 1. Cross-sections of three-layer microstrip-like transmission lines

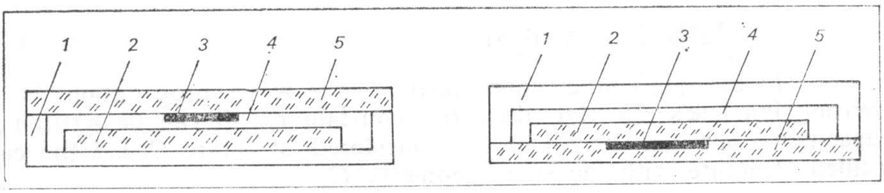

Fig. 2. Possible application of three-layer microstrip-like transmission lines to nondestructive measurement of dielectric sheets (*1* — metal test structure, *2* — substrate under test, *3* — metal strip conductor, *4* — air gap, *5* — microstrip substrate with known permittivity)

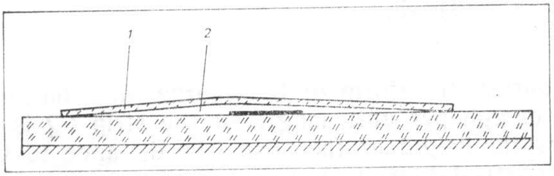

Fig. 3. Air gap by nondestructive measurement of dielectric substrates due to the curvature of the measured substrate (*1* — measured substrate, *2* — air gap)

structure is obtained by putting a dielectric plate over an usual microstrip line by the permittivity measurement method presented in [1]. They are due to the substrate curvature (Fig. 3) and set limits to the measurement accuracy.

The first example shows that the method must be fast enough, because the matter of interest is not the analysis itself, but the inverse problem, i. e. computing the unknown permittivity ε_x from the known propagation constant. Solving this problem requires the analysis procedure to be repeated many times changing ε_x until the computed and measured guided wavelengths become equal. In microstrip measurement technique the error due to the inaccurate values for the line dimensions and material permittivities is about 1—3 per cent. Therefore one can accept methods ensuring such accuracy if they are fast enough even when more accurate, but slower methods exist. An important consequence therefore is that in most of the cases the quasi-static approximation must be preferred to the full electrodynamic analysis. The accuracy of this approximation is about 1—2 per cent for lower frequencies when the wavelength is much greater than the sheet thicknesses and the strip conductor width.

The problem of wave propagation along a microstrip-like transmission line with any number of dielectric sheets and metal strips has been studied in [2]. The methods suggested there are very complicated to be realized on small desk-top computers because of insufficient computation speed. Even if realized, they are unnecessarily complicated and in the case of three-layer structures easier procedures can be applied.

This paper presents such a method. It is an extension of the variational method for the analysis of microstrip-like transmission lines with one and two dielectric sheets [3, 4]. Three and not one or two dielectric sheets are considered and a new charge distribution treatment is suggested to increase the accuracy. The charge distribution $f(x)$ is expanded in terms of known basic functions containing the edge singularities, instead of using the probe function $1+|x/2w|^3$ customary to variational method analysis, as made earlier [4].

2. Method of analysis

In the quasistatic approximation the wave propagation is described by the transmission line's effective dielectric constant ε_{eff}. It can be written in terms of the capacitance per unit length of the considered structure C and the capacitance of an air-filled structure with the same geometry C_0

$$\varepsilon_{\text{eff}} = C/C_0. \tag{1}$$

The procedure to compute C will be discussed in details further, as C_0 is a special and easier case when all permittivities are equal to unity.

The capacitance per unit length C is computed using the variational expression

$$\frac{1}{C} = \frac{1}{\pi\varepsilon_0 Q^2}\int_0^{+\infty} G(\beta\,;\,h,\,h)[\tilde{f}(\beta)]^2 d\beta, \tag{2}$$

where $\tilde{f}(\beta)$ is the Fourier transform of the charge distribution over the metal strip $f(x)$, Q is the total charge per unit length of the strip (Q is set to one for normalization purposes), h is the height of the strip (the distance to the grounded plane) and $G(\beta;\, y_1,\, y_2)$ is the spectral-domain Green's function. It is defined as the amplitude of the space harmonic with wave number β in the horizontal potential distribution at height y_2 due to the same space harmonic in a charge distribution at height y_1.

$G(\beta;\, h,\, h)$ is computed in the standard way [4] by solving four ordinary second order differential equations in the separate homogeneous areas and applying the appropriate boundary conditions at the dielectric interfaces.

For the structure from Fig. 1*b* $h=h_1+h_2$ and

$$\frac{1}{\beta G} = \varepsilon_2\,\frac{\varepsilon_1+\varepsilon_2\tanh\beta h_1\tanh\beta h_2}{\varepsilon_1\tanh\beta h_2+\varepsilon_2\tanh\beta h_1} + \varepsilon_3\,\frac{\varepsilon_3\tanh\beta h_3+\varepsilon_4}{\varepsilon_3+\varepsilon_4\tanh\beta h_3}. \tag{3}$$

For the structure from Fig. 1*c* $h=h_1$ and

$$\frac{1}{\beta G} = \varepsilon_1\operatorname{cotanh}\beta h_1 + \varepsilon_2\,\frac{\varepsilon_2\tanh\beta h_2+\varepsilon_3\,\dfrac{\varepsilon_3\tanh\beta h_3+\varepsilon_4}{\varepsilon_3+\varepsilon_4\tanh\beta h_3}}{\varepsilon_2+\varepsilon_3\,\dfrac{\varepsilon_3\tanh\beta h_3+\varepsilon_4}{\varepsilon_3+\varepsilon_4\tanh\beta h_3}\tanh\beta h_2}. \tag{4}$$

The structure from Fig. 1*a* is not of practical interest and will not be presented here.

After having found the expressions for G in both cases, some procedure must be used to get an expression for $\tilde{f}(\beta)$ or $f(x)$. The standard method [3] is to use some test function and hope that the result obtained is close enough to the real value of C because (4) has a flat minimum around the real distribution $\tilde{f}(\beta)$. The most popular choice is [5]

$$f(x)=\begin{cases}1+|2x/w|^3 & \text{for } |x|<w/2\\ 0 & \text{for } |x|\geq w/2.\end{cases} \tag{5}$$

This choice is not good enough due to two reasons.

Unlike (5) the real distribution changes for different values of w/h. In the case $w\gg h$ it is almost uniform over the greater part of the strip conductor and increases only near the strip edges at distances of about $2h$. In order to replace (5) with a more realistic distribution and not to use a priori any information about $f(x)$ it can be written as the sum of known basis functions $f_n(x)$ with unknown coefficients a_n to be found later

$$f(x)=\sum_{n=1}^{N} a_n f_n(x), \tag{6}$$

$$\tilde{f}(\beta)=\sum_{n=1}^{N} a_n \tilde{f}_n(\beta), \tag{7}$$

$$\tilde{f}_n(\beta)=\int_{-w/2}^{+w/2} f_n(x)\, e^{j\beta x} dx. \tag{8}$$

Increasing N any desired accuracy can be achieved. Then (2) takes the form:

$$\frac{1}{C}=\frac{1}{\pi\varepsilon_0 Q^2}\sum_{n=1}^{N}\sum_{m=1}^{N} G_{nm} a_n a_m, \tag{9}$$

where

$$G_{nm}=\int_{0}^{+\infty} G(\beta;\, h,\, h)\tilde{f}_n(\beta)\tilde{f}_m(\beta)\, d\beta, \quad n,\, m=1,\, 2,\, \ldots,\, N. \tag{10}$$

The normalizing condition $Q=1$ takes the form:

$$\sum_{n=1}^{N} a_n I_n = 1, \tag{11}$$

where I_n are the integrals

$$I_n=\int_{-w/2}^{+w/2} f_n(x)\, dx=\tilde{f}_n(0). \tag{12}$$

Minimizing (9) under condition (11) leads to a linear system for the unknown coefficients a_n, $n=1,\, 2,\, \ldots,\, N$ and the Lagrange's multiplier λ

$$\begin{pmatrix} 2G_{11} & 2G_{12} & \dots & 2G_{1N} & I_1 \\ 2G_{21} & 2G_{22} & \dots & 2G_{2N} & I_2 \\ \dots & \dots & \dots & \dots & \dots \\ 2G_{N1} & 2G_{N2} & \dots & 2G_{NN} & I_N \\ I_1 & I_2 & \dots & I_N & 0 \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \\ \dots \\ a_N \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \dots \\ 0 \\ 1 \end{pmatrix} \qquad (17)$$

which is easily solved.

The second disadvantage of the distribution (5) is that it does not take into account the strong singularity at the strip edges which is of the kind

$$f(x)\,\alpha\,\frac{1}{\sqrt{|x-x_{\text{edge}}|}}\,. \qquad (14)$$

Using basis functions that do not take into account this fact leads to less realistic charge distributions and worse convergence. Therefore the basis functions should behave like (14) near the strip edges. For convenience their Fourier transforms (8) should be closed form expressions. To meet these requirements the following basis functions were chosen:

$$f_n(x) = \frac{T_{2n}(2x/w)}{\sqrt{1-(2x/w)^2}}, \quad n=1, 2, \dots, N \qquad (15)$$

with the Fourier transforms

$$\tilde{f}_n(\beta) = (-1)^n \ (w/2)\pi J_{2n}(\beta w/2) \qquad (16)$$

and

$$I_n = \pi \ (w/2)\ \delta_{0n}. \qquad (17)$$

Here T_m stands for a Chebychev's polinomial of order m, J_m is a Bessel function of order m, and δ_{mn} is Kronecker delta symbol.

The method presented is in some sense near to the Galerkin's method in spectral-domain full electrodynamic analysis, as the main computation effort is connected with computing the inner products of the Fourier transforms of the basis functions with weight function $G(\beta;\, h,\, h)$. The difference is that this is done only once, unlike the full electrodynamic case, where all integrals are computed for each value of the longitudinal wave number k_z in the process of solving a transcendental equation of the kind $\det M(\omega,\, k_z)=0$. This is the source of reducing the computing time.

3. Numerical results

This computation method was applied in a desk-top computer program. Figure 4 shows some computed charge distributions compared with (5). It is evident that in the case of $w \gg h$ (5) is no longer realistic enough.

Figure 5 shows the effective dielectric constants of some transmission lines with three dielectric interfaces, that can be of interest for nondestructive permittivity measurement.

4. Analysis of the sensitivity

The quantity determining the possibility to use certain microstrip-like transmission line for nondestructive permittivity measurement is the sensitivity of the effective dielectric constant ε_{eff} to the unknown permittivity ε_x defined as follows:

$$S_{\varepsilon_x}^{\varepsilon_{\text{eff}}} = \frac{\varepsilon_x}{\varepsilon_{\text{eff}}} \cdot \frac{\partial \varepsilon_{\text{eff}}}{\partial \varepsilon_x}\,. \qquad (18)$$

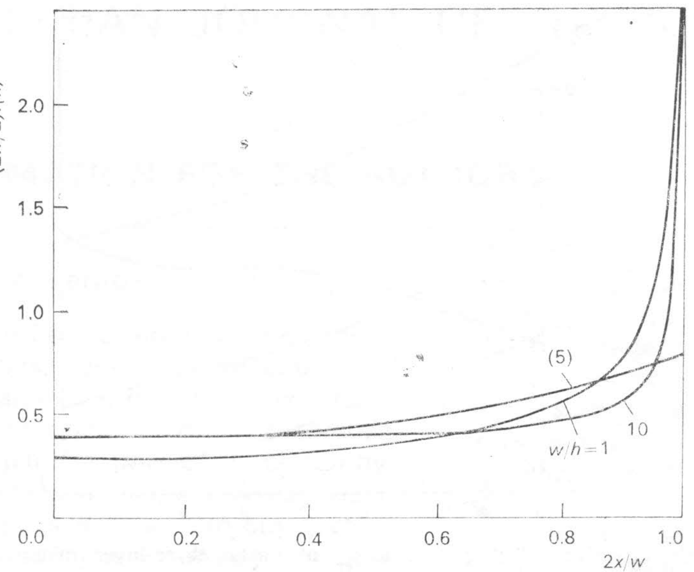

Fig. 4. Computed charge distributions in a three-layer microstrip-like transmission line ($h_1/h_2=4.0$, $h_3/h_2=3.0$, $h=h_1+h_2$; $\varepsilon_1=1.0$, $\varepsilon_2=2.5$, $\varepsilon_3=2.0$, $\varepsilon_4=1.0$)

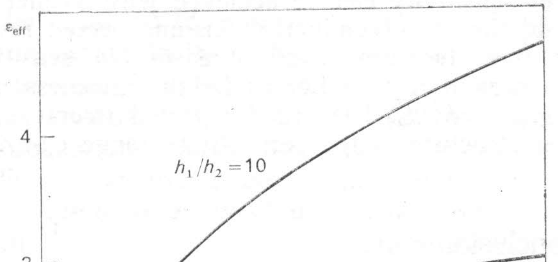

Fig. 5. Effective permittivity for wide microstrip-like transmission lines with three dielectric interfaces ($h_1/w=2.0$, $h_3/w=1.2$; $\varepsilon_2=1.0$, $\varepsilon_3=2.5$, $\varepsilon_4=1.0$)

In Fig. 6 this quantity is plotted against ε_x for several cases that are of practical interest. The sensitivity of the standard two-layer structure from [1] is plotted for comparison.

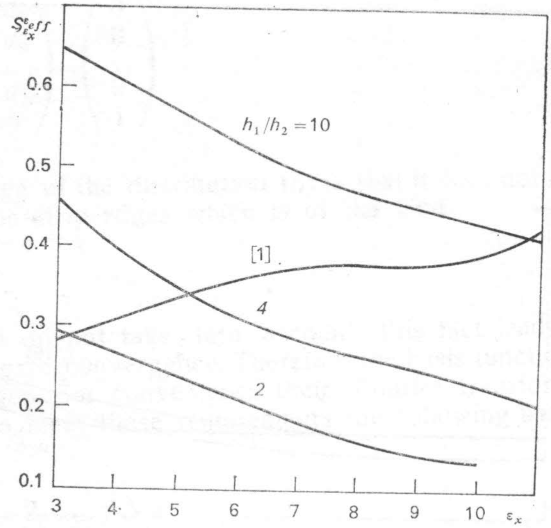

Fig. 6. Sensitivity of the effective dielectric constant ε_{eff} of a wide three-layer microstrip-like transmission line ($h_1/w=2.0$, $h_3/w=1.2$; $\varepsilon_2=1.0$, $\varepsilon_1=\varepsilon_x$, $\varepsilon_3=2.5$, $\varepsilon_4=1.0$) to the unknown permittivity ε_x compared with the sensitivity of a standard two-layer structure ($h_1/w=2.0$, $h_2=0$, $h_3/w=1.2$; $\varepsilon_1=2.5$, $\varepsilon_3=\varepsilon_x$, $\varepsilon_4=1.0$)

As expected, better sensitivity can be achieved with smaller air gaps between the substrate under test and the strip conductor. An unexpected result is that in case of wide strip conductor this structure leads to reasonable sensitivities only for small values of the unknown permittivity ε_x. Let us fix the thickness ratio of the measured substrate and the air gap $r=h_1/h_2$. Then the suggested structure shows higher sensitivity than the two layer structure in the permittivity range $\varepsilon_x<r$.

5. Conclusion

The good sensitivity and the existence of a fast analysis method makes the microstrip-like transmission lines with three dielectric interfaces applicable to non-destructive permittivity measurements of dielectric sheets.

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