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INTERACTION IN PARTICLE-PARTICLE CHANNEL AND ANHARMONICITY OF SURFACE VIBRATIONS IN SPHERICAL NUCLEI

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Abstract. The influence of the interaction in the particle-particle channel on the properties of low-lying excited states (2_1^+, 2_2^+, 4_1^+, 4_2^+) in some spherical nuclei (^{122}Te, ^{114}Cd, ^{116}Cd) is studied. The studies are carried out in the framework of the quasiparticle-phonon model. The results point out that the switching cn of the interaction decreases the anharmonic quasiparticle-phonon matrix element and it increases the value of the two-phonon components in the structure of the wave function of 2_2^+ and 4_1^+ states. It is in agreement with the experimental data. More adequate description of the properties of low-lying states could be reached by diagonalizing the Hamiltonian in a larger basis including three-phonon components.

Резюме. Исследовано влияние взаимодействия в канале частица-частица на свойства низколежащих состояний (2_1^+, 2_2^+, 4_1^+, 4_2^+) некоторых сферических ядер (^{122}Te, ^{114}Cd, ^{116}Cd). Исследование проводилось в рамках квазичастично-фононной модели атомного ядра. Результаты показали, что включение взаимодействия приводит к уменьшению ангармонического квазичастично-фононного матричного элемента, и поэтому вклад двухфононной компоненты в структуру волновой функции 2_2^+- и 4_1^+-состояний увеличивается. Это приводит к улучшению согласия с экспериментальными данными. Более точного описания свойств низколежащих состояний можно достичь, проводя диагонализацию Гамильтониана на более широком базисе, включая трехфононные компоненты.

1. Introduction

The interaction in the particle-particle channel has been studied by many authors [1—8]. It is shown that its importance is predominantly for the description of two-nucleon transfer reaction [4] and at high excitation energies [8]. Recently, the interest in that interaction has been revived in connection with the problem of the description of double β decay [9, 10]. Also, it is shown that the interaction influences the Gamow-Teller β decay [10].

In this paper the interaction in the particle-particle channel is studied as part of the general problem for a more precise description of the properties of low-lying excited states in spherical nuclei. It is well known [11] that the anharmonic vibrator is a good basis for the description of the characteristics of the first few excited states in spherical nuclei. The microscopic mapping of an anharmonic vibrator is connected mainly with the particle-hole channel of the residual two-nucleon interaction [11, 12]. The purpose of the presented investigation is to estimate the influence of the particle-particle channel of the interaction on the properties of an anharmonic vibrator. Numerical results are given for some Cd and Te isotopes.

2. Formal results

The interaction in the particle-particle channel could be included in the nuclear Hamiltonian as a separable multipole term [13]:

$$H_{M}^{pp}=1/2\sum_{\lambda\mu}[G_{nn}^{(\lambda)}P_{\lambda\mu}^{+}(n)P_{\lambda\mu}(n)+G_{pp}^{(\lambda)}P_{\lambda\mu}^{+}(p)P_{\lambda\mu}(p) \tag{1}$$

$$+G_{np}^{(\lambda)}[P_{\lambda\mu}^{+}(n)P_{\lambda\mu}(p)+P_{\lambda\mu}^{+}(p)P_{\lambda\mu}(n)]].$$

Equation (1) together with the well known terms of mean field, pairing forces and multipole interaction in a particle-hole channel yields a more general Hamiltonian of the microscopic model based on superfluid and multipole-multipole separable residual interaction, i. e.

$$H=H_{av}+H_{pair}+H_{M}$$

$$H_{M}=H_{M}^{ph}+H_{M}^{pp}.$$

The operator P^{+} given in eq. (1) has the form:

$$P_{\lambda\mu}^{+}(\tau)=\sum_{jj'}^{\tau}\sum_{mm'}F_{jm,j'm'}^{(\lambda\mu)}(-)^{j'-m'}a_{jm}^{+}a_{j'-m'}^{+}, \tag{2}$$

where a_{jm}^{+} is a particle creation operator having quantum number j (j assumes quantum numbers n, l, j) and its projection m. The matrix element $F_{jm,j'm'}^{(\lambda\mu)}$ is the single-particle matrix element of the residual separable interaction. The parameters $G_{\tau\tau}^{(\lambda)}$ are strength parameters.

The diagonalization of the Hamiltonian can be carried out very easily in the frame of the RPA method. The corresponding phonon basis reads:

$$Q_{\lambda\mu i}^{+}=\sum_{j_1j_2}\{\Psi_{j_1j_2}^{\lambda i}[\alpha_{j_1m_1}^{+}\alpha_{j_2m_2}^{+}]_{\lambda\mu}-(-1)^{\lambda-\mu}\Phi_{j_1j_2}^{\lambda i}[\alpha_{j_1m_1}\alpha_{j_2m_2}]_{\lambda\mu}-\mu\}. \tag{3}$$

Here α_{jm} is the quasiparticle creation operator, curly brackets stand for angular momentum coupling, Ψ and Φ are quasiparticle amplitudes.

The equations for the energy of the excited states as well as the equations for the amplitudes Ψ and Φ are given in refs [8, 15].

To take into consideration the correlation between nucleons beyond the RPA it is helpful to use a well known nuclear model. We have used the Quasiparticle Phonon Model (QPM) [14]. In the framework of QPM the anharmonic effects are described as an interaction between quasiparticles and phonons. Taking into account eq. (2) and following the prescription in ref. [14] it is easy to obtain the next equation for the quasiparticle-phonon interaction:

$$H_{qpph}=-1/2\sqrt{2}\left\{\sum_{\lambda\mu i}(-1)^{\lambda-\mu}\sum_{j_1j_2}K[^{(+)}(j_1j_2\lambda i)\,Q_{\lambda\mu i}^{+}\right.$$

$$+(-1)^{\lambda-\mu}K^{(-)}(j_1 j_2 \lambda\mu)Q_{\lambda-\mu i}]\,B(j_1 j_2 \lambda -\mu)+\text{h. c.}\Big\}. \tag{4}$$

Here

$$K^{(\pm)}(j_1 j_2 \lambda i)=f^{(\lambda)}_{j_1 j_2}/\sqrt{y_{\lambda i}\,[v^{(-)}_{j_1 j_2}\mathcal{R}^{(\lambda i)}-2u_{j_2}v_{j_1}(\mathfrak{L}^{(\lambda i)}\pm\mathcal{M}^{(\lambda i)})]}, \tag{5}$$

where $f^{(\lambda)}_{j_1 j_2}$ is reduced single-particle matrix element, $v^{(-)}_{j_1 j_2}$, u_j and v_j are the Bogolubov coefficients and combinations of them [12], the eqs for $y_{\lambda i}$, $\mathcal{R}^{(\lambda i)}$, $\mathfrak{L}^{(\lambda i)}$, $\mathcal{M}^{(\lambda i)}$ are complicated and are given in ref. [8].

The operator $B(j_1 j_2 \lambda\mu)$ has the form:

$$B(j_1 j_2 \lambda\mu)=[\alpha^+_{j_1 m_1}\alpha_{j_2 m_2}]_{\lambda\mu}. \tag{6}$$

The interaction (4) together with the RPA part of the Hamiltonian forms the QPM Hamiltonian [13, 14]

$$H=H_{\text{RPA}}+H_{\text{qpph}}.$$

The above Hamiltonian is diagonalized in the following basis:

$$\Psi(JM)=\Big\{\sum_i R_i\,Q^+_{\text{JM}}+\sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}} P^{\lambda_1 i_1}_{\lambda_2 i_2}[Q^+_{\lambda_1\mu_2 i_1}\,Q^+_{\lambda_2\mu_2 i_2}]_{\text{JM}}\,|0>\Big\}. \tag{7}$$

The new point in the formal results in comparison with the widespread version of QPM [14] is the difference in the value of the matrix element between one- and two-phonon states, i. e.

$$\langle Q_{\text{JM}i}\,|\,H_{\text{qpph}}\,|\,[Q^+_{\lambda_1\mu_1 i_1}\,Q^+_{\lambda_2\mu_2 i_2}]_{\text{JM}}\rangle. \tag{8}$$

The difference comes from the different equations for the values of $K^{(\pm)}(j_1 j_2 \lambda i)$. Only the first part of the sum (5) is taken into account in QPM, i. e.

$$K_{\text{QPM}}(j_1 j_2 \lambda\mu)=f^{(\lambda)}_{j_1 j_2}/\sqrt{y_{\lambda i}\,v^{(-)}_{j_1 j_2}\mathcal{R}^{(\lambda i)}}$$

and eq. (8) reads

$$\langle Q_{\text{JM}i}\,|\,H_{\text{qpph}}\,|\,[Q^+_{\lambda_1\mu_1 i_1}\,Q^+_{\lambda_2\mu_2 i_2}]_{\text{JM}}\rangle \tag{9}$$

$$=U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)_{\text{QPM}}\equiv U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)_{\text{R}}.$$

Obviously the switching on of the interaction in the particle-particle channel the matrix element (8) reads:

$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)=U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)_{\mathcal{R}}+U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)_{\mathcal{L}}\pm U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)_{\mathcal{M}}. \tag{10}$$

The equation for $U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$ in the particle-hole channel is given in ref. [15]. The same equation is obtained in the present study for each term included in (10).

3. Numerical results

To test numerically the influence of the particle-particle channel some Cd and Te isotopes have been chosen. These nuclei are spherical having remarkable admixture of noncollective components in the "two-phonon" states [16]. The standard set of the model parameters has been used in the calculations [13, 16]. The values of the new parameters coming from the particle-particle channel have been chosen following the condition:

$$G^{(\lambda)}_{nn} = G^{(\lambda)}_{pp} = G^{(\lambda)}_{np} = G^{(\lambda)}. \tag{11}$$

The most interesting is the quadrupole case, i. e. $\lambda = 2$. The value ot $G^{(2)}$ could be fixed in RPA calculating the energy of the first 2^+ state and the $B(E2, 0^+_{g.s.} \rightarrow 2^+_1)$. The next rule is used: to fit the calculated energy of the 2^+_1 state on the experimental value at the maximum of $B(E2)$. Additionally the value of isoscalar parameter $\varkappa^{(2)}_0$ could be slightly changed to reduce the strength of the quasiparticle-phonon interaction (4). The value of $G^{(\lambda)}$ $(\lambda \neq 2)$ is taken to be zero.

It is useful to estimate the influence of the terms $U_{\mathscr{L}}$ and $U_{\mathscr{M}}$ on the anharmonic matrix element (8, 10). The results are given in Table 1. It is seen that the value of $\mathscr{L}$ and $\mathscr{M}$ are at least 10 times less than $\mathscr{R}$ and it means that $U_{\mathscr{L}}$ and $U_{\mathscr{M}}$ could be neglected in (10). So, the different values of the matrix elements (10) calculated in QPM and those of the present study are due to the different values of the RPA amplitudes Ψ and Φ (see eq. 3). The amplitudes Ψ and Φ are an important part of the matrix element (10) [15].

Table 1. The value of $\mathscr{R}$, $\mathscr{L}$ and $\mathscr{M}$ for the 2^+_1-state

	$\mathscr{R}$	$\mathscr{L}$	$\mathscr{M}$
^{122}Te	0.276×10^{-1}	0.312×10^{-2}	0.342×10^{-2}
^{114}Cd	0.279×10^{-1}	0.194×10^{-4}	0.530×10^{-2}
^{116}Cd	0.271×10^{-1}	-0.287×10^{-3}	0.510×10^{-2}

Table 2. Properties of low-lying states. Coefficients R and P are defined in eq. (7). Index *max* means the largest coefficients in the structure of the state Experimental values are taken from ref. [17]

Table 2*a*. Only the particle-hole channel is taken in.o account

Nucleus	J^π	*p - h* E, MeV	Structure max R^2	max P^2	$B(E2\uparrow)$ e^2b^2 $e_{eff}=0.2$	$U_{2^+_1 2^+_1}(J^\pi)$, MeV
^{122}Te	2^+_1	0.584	86%	2%	0.619	0.272
	2^+_2	1 744	23%	7%	—	—
	4^+_1	1.43	23%	56%	—	0.60
^{114}Cd	2^+_1	0.565	75%	17%	0.325	1.30
	2^+_2	2.57	59%	14%	—	—
	4^+_1	1.56	64%	27%	—	—0.84
	4^+_2	2.65	17%	46%	—	—
^{116}Cd	2^+_1	0.520	74%	18%	0.327	1.28
	2^+_2	2.46	51%	16%		—
	4^+_1	1.49	62%	27%	—	—0.81
	4^+_2	2.53	19%	30%	—	—

Table 2b. Both channels are taken into account

Nucleus	J^π	*p-h+p-p*			$B(E2)\ e^2b^2$	$U\ \frac{2_1^+}{2_1^+}(J^\pi)$,	Exp.	
		E, MeV	Structure		$e_{eff}=0.3$	MeV	E, MeV	$B(E2)\ e^2b^2$
			max R^2	max P^2				
^{122}Te	2_1^+	0.568	91%	3%	0.723	−0.06	0.564	0.66
	2_2^+	1.614	2.7%	94%	—	—	1.257	
	4_1^+	1.33	11%	76%	—	0.45		
^{114}Cd	2_1^+	0.543	77%	15%	0.365	0.95	0.558	0.51
	2_2^+	2.18	24%	54%	—	—	1.209	—
	4_1^+	1.50	49%	42%	—	−0.69	1.283	—
	4_2^+	2.36	39%	46%	—	—	—	—
^{116}Cd	2_1^+	0.509	77%	17%	0.357	0.95	0.514	0.55
	2_2^+	2.131	22%	56%	—	—	1.214	—
	4_1^+	1.46	47%	43%	—	−0.7	1.22	–
	4_2^+	2.32	34%	44%				

Table 3. The value of the reduced transition probabilities between the 2_1^+ and the 2_2^+ excited states. Experimental values are taken from ref. [17]. Effective charge e^* is 0.0 and 0.3

$B(E2,\ 2_1^+ \longrightarrow 2_2^+)e^2\ b^2$

	p-h		*p-h+pp*		Exp.
	$e^*=0.0$	$e^*=0.3$	$e^*=0.0$	$e^*=0.3$	
^{122}Te	0.011	0.03	0.09	0.277	
^{114}Cd	0 003	0.01	0,016	0.051	0.07
^{116}Cd	0.004	0.015	0.017	0.057	

The results for the energy and the sructure of the 2_1^+ and the 2_2^+ states are given in Table 2. We would like to point out the following features:

1. The collectivity of the 2_1^+ state decreases due to the particle-particle interaction. That is seen from Table 2 as the higher value of the effective charge is used to reach approximatelly the same value of $B(E2\uparrow)$. For cadmium isotopes $e_{off}=0.3$ is not sufficient to reach the experimental value of $B(E2)$).

2. The value of the matrix element (8), i. e. the strength of anharmonicity decreases due to the particle-particle interaction.

3. The structure of the first 2^+ state is conserved while the value of the two-phonon component (denoted max P^2 in the table — P^2 means the contribution of the two-phonon component in the norm of the wave function (7) in per cent) in 2_2^+ state increases much (more than 3 times) due to the particle-particle interaction.

4. The energy of 2_2^+ state is less in the case when the particle-particle interaction is switched on but it is still far from the experimental value.

5. The same conclusions (points 2—4) are valid for the 4^+ states given in the Table.

The drastic change in the two-phonon component in the 2_2^+ and 4^+ states increases the $B(E2)$ values between the 2_1^+ and 2_2^+ states (see Table 3). It is in agreement with the experimental data.

4. Conclusions

The results presented in this paper lead us to the general conclusion that the influence of the interaction in the particle-particle channel on the properties of the 2_2^+ and the lowest 4^+ states in spherical nuclei is remarkable. It means that in more complex calculations where the goal is a precise description of the structure of low-lying states, the interaction in the particle-particle channel must be taken into account.

It is seen, on the other hand, that it is not possible to reach the realistic description of the structure of low-lying states including in the Hamiltonian only the term (4). A more profound approach could be found combining the effect discussed in this paper with more complex basis including three-phonon components [18].

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