BULGARIAN JOURNAL OF PHYSICS
volume 17 1990 number 4

0400

THE G. U. T. MASS SCALE FROM THERMODYNAMIC REASONING

C. Wolf

Department of Physics, North Adams State College, North Adams, MA (01247) U.S.A.

Abstract. By considering an ensemble of nonrelativistic quantum particles living in a superposition of dimensionality up to N_0, we demonstrate that if in addition superheavy Planck mass particles are generated by an unpaired twofold quantum variable, that the most favorable thermodynamic configuration is for 10^{63} quantum particles living in four space time dimensions with no superheavy Planck mass particles generated at the Planck scale. Our argument favours these heavy G. U. T. scale particles as constituents of the early universe.

Резюме. В статье рассматривается энтропия спиновой системы в пространстве, размерность которого может варьироваться. Из нахождения максимума энтропии как функции числа частиц и их кинетической энергии высказываются соображения о том, почему размерность пространства-времени должна быть $3+1$, а число частиц в ранней Вселенной 10^{63} для диаметра Вселенной 10^{10} cm. При общей массе Вселенной 10^{54} g для масс частиц получается величина 10^{-9} g, что по порядку оценкам массы калибровочных бозонов и теории Великого объединения.

*Переведено редакцией

1. Introduction

The earliest stages of cosmological evolution have been thought of as deeply connected to the mysteries of quantum gravity along with our uncertain knowledge of a correct G. U. T. theory at high energy [1]. The heterotic string has given us a fresh attack to the problem of cosmic evolution by presenting us with an effective supergravity cosmology in ten dimension that may provide us with a correct mechanism for compactification of extra dimensions through Calabi Yau spaces [2]. However, this approach is still in question because of the uncertain path taken by the compactification process and the mathematical complexity of Calabi Yay space [3]. To circumvent this problem we have suggested a statistical scenario for the big bang based on the collapse of a spin system generating the inertia of the initial expansion by the disordering of the spins [4]. We later expanded our ideas to a theory of pregeometry by constructing a lattice model out of the primitive twofold quantum variables with harmonic oscillators connecting the junctions [5]. In this note we consider an extension of the ideas in refs [4, 5] by considering j nonrelativistic particles living in a superposition of dimensionality up to N_0. We also consider, in addition, Planck mass particles of superheavy mass generated by an unpaired twofold quantum variable. By maximizing the entropy of the combined system of twofold quantium variables and

j nonrelativistic particles moving in a superposition of dimensionality up to N_0, we find that 3 is the most favorable space dimensionality and also there are no Planck mass particles generated. We also find that the number of quantum particles, j, is very large, but less than the present number of protons in the present universe which suggests that the early universe consisted of G. U. T. mass scale particles. Though our treatment is nonrelativistic, we view it as a model calculation for a full relativistic calculation where at least the qualitative features of our discussion will be the same.

2. Fragmentation model of early universe coupled to a spin system representing a twofold quantum variable

We consider a spin system (system of twofold variables) coupled to a system of quantum particles in 1, 2; N_0 dimensions, where we will superimpose a j body break-up in each dimension and add up the phase spaces multiplicitivity to N_0. For the degeneracy function of a system with j body break-up and a spin system with $N/2+m$ spin up, $N/2-m$ spin down, we have

$$g=g(N,0)e^{-\frac{2m^2}{N}}\prod_{D=1}^{N_0}\left(\frac{\pi}{4}\right)^j\left(\frac{8ML^2}{jh^2}\right)^{\frac{D}{2}j}\left(\frac{E}{j}\right)^{j\left(\frac{D}{2}-1\right)}\left(\frac{1}{j}\right)^j(\delta E)^j, \tag{2.1}$$

where $g(N,0)\,e^{-\frac{2m^2}{N}}$ represents the number of ways of realizing $\frac{N}{2}+m$ spin up, and $\frac{N}{2}-m$ spin down quantum variables. The second term for D dimensions comes from the volume of a quadrant in D dimensions $\frac{\pi n^D}{2D}$

$$\text{with } \frac{E}{j}=n^2h^2/8(M/j)L^2. \tag{2.2}$$

Here L is the length scale of universe, M is the rest mass of j non-Planck scale particle in universe, $n=\left(\frac{8ML^2}{jh^2}\right)^{\frac{1}{2}}\left(\frac{E}{j}\right)^{\frac{1}{2}}$, giving $V=\frac{\pi}{2D}\left(\frac{8ML^2}{jh^2}\right)^{\frac{D}{2}}\left(\frac{E}{j}\right)^{\frac{D}{2}}$ with $\delta V=\frac{\pi}{4}\left(\frac{8ML^2}{jh^2}\right)^{\frac{D}{2}}\left(\frac{E}{j}\right)^{\frac{D}{2}-1}\left(\frac{1}{j}\,\delta E\right)$, then multiplying out the j terms, we have for j bodies as above in Eq. (2.1)

$$(\delta V_{\mathrm{Tot}})_D=\left(\frac{\pi}{4}\right)^j\left(\frac{8ML^2}{jh^2}\right)^{\frac{Dj}{2}}\left(\frac{E}{j}\right)^{\left(\frac{D}{2}-1\right)j}\left(\frac{1}{j}\right)^j\delta E^j. \tag{2.3}$$

For the energy conservation principle we have

$$2m\mu_0+E=E_0, \tag{2.4}$$

where $2m\mu_0$ is the energy of unpaired quantum variables and represents the rest energy of Planck scale particles and we neglect KE of Planck scale particles; E is the kinetic energy of j nonrelativistic non-Planck scale particles, and E_0 is the total energy of universe — rest energy of j non-Planck scale particles. Taking the natural log of Eq. (2.1), we have

$$\ln_e g = \ln_e g(N, 0) - \frac{2}{N}\left(\frac{E_0 - E}{2\mu_0}\right)^2 + \sum_{D=1}^{N_0} j \ln_e \frac{\pi}{4} + \sum_{D=1}^{N_0} \frac{Dj}{2} \ln_e\left(\frac{8ML^2}{jh^2}\right)$$

$$+ \sum_{D=1}^{N_0} j\left(\frac{D}{2} - 1\right) \ln_e \frac{E}{j} - \sum_{D=1}^{N_0} j \ln_e j + \sum_{D=1}^{N_0} j \ln_e \delta E$$

or

$$\ln_e g = \ln_e g(N, 0) - \frac{2}{N}\left(\frac{E_0 - E}{2\mu_0}\right)^2 + N_0 j \ln_e \frac{\pi}{4} + \frac{j}{2}\left(\frac{N_0(N_0+1)}{2}\right) \ln_e \frac{8ML^2}{jh^2} \tag{2.5}$$

$$+\left(\frac{N_0(N_0+1)}{4} - N_0\right) j \ln_e \frac{E}{j} - N_0 j \ln_e j$$

after setting $\sum_{D=1}^{N^0} j \ln_e \delta E \approx 0.$

Now we maximize the log of the degeneracy function, which is maximizing the entropy; the independent variables are E and j. From Eq. (2.5), we have

$$\frac{d(\ln_e g)}{dE} = 0, \quad \frac{d(\ln_e g)}{dj} = 0, \tag{2.6}$$

giving

$$\frac{(E_0 - E)}{N\mu_0^2} + \left[\frac{N_0(N_0+1)}{4} - N_0\right]^{\frac{j}{E}} = 0, \tag{2.7}$$

$$N_0 \ln_e \frac{\pi}{4} + \frac{N_0(N_0+1)}{4}\left[\ln_e \frac{8ML^2}{h^2 j} - 1\right] + \left[\frac{N_0(N_0+1)}{4} - N_0\right] \ln_e \frac{E}{j} \tag{2.8}$$

$$-\left[\frac{N_0(N_0+1)}{4} - N_0\right] - N_0 \ln_e j - N_0 = 0.$$

Equation (2.7) gives

$$E = \frac{E_0 \pm \sqrt{E_0^2 + 4N\mu_0^2 j(N_0(N_0+1)/4 - N_0)}}{2}. \tag{2.9}$$

For $N_0 = 3$, $E = 0$, or E_0; for $N > 3$, $E > E_0$ or $E < 0$, which gives negative KE of the light particles or negative mass for the Planck scale particles from Eq. (2.4). Both cases for $N_0 > 3$ are unphysical. For $N_0 = 3$, we find that $E = E_0$, gives maximum entropy which, from Eq. (2.4), gives $m = 0$ or 0 mass for the Planck scale particles. From Eq. (2.8) we have neglecting

$$N_0 \ln_e \frac{\pi}{4} \simeq 0,$$

for $N_0 = 3$

$$\ln_e \frac{8ML^2}{h^2 j^2} \simeq 2; \quad j^2 \simeq \frac{ML^2}{h^2}. \tag{2.10}$$

Here, $M \simeq 10^{54}$ grams, $L \simeq 10^{10}$ cm, j is 10^{63} or the mass of each particle is 10^{-9} grams. This is about the mass of a heavy G. U. T. gauge boson. The above calculation is based on the assumption that the scale of the early universe is about 10^{10} cm. We also

Table 1. Variation of j with L

L, cm	j	M_X, grams
1	10^{53}	10
10^4	10^{57}	10^{-3}
10^{10}	10^{63}	10^{-9}
10^{14}	10^{67}	10^{-13}

put in 10^{54} for the known present rest mass of the universe. In the following Table we study the variation of j with L.

3. Conclusion

From the above simplified calculation we see that $N_0=3$, or 3 space dimensionality is favoured to prevent negative mass Planck scale particles generated by the above unpaired quantum variable. We also note that the mass of a single non-relativistic particle is about the mass of a G. U. T. gauge boson if the scale of the universe is about 10^{10} cm. This calculation has assumed no binding between the quantum particles and essentially assumes that gravity is asymptotically free at these scales. The fact that we have arrived at such attractive numbers by avoiding any complicated discussion of particle theory makes it all the more the reason to pursue this simple model further.

Acknowledgements. I would like to thank the Physics departments at Williams College and Harvard University for the use of their facilities.

References

1. Vilenkin, A. — Proc. of J. Hopkins Workshop on Current Problems in Particle Theory (8), World Scientific, Baltimore, Singapore, June 20—22, 1984.
2. Moffat, J. W. — Can. J. Phys., 64, 1986, p. 651.
3. Strominger, A. — Nucl. Phys., B 274, 1986, p. 253.
4. Wolf, C. — Int. J. of Theoretical Phys., 25, 1986, 12, p. 1229.
5. Wolf, C. (to appear in Foundations of Physics, 1990).

Received December 11, 1989