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On Enceladian Fields

In tribute to Murray Gell-Mann

T.S. Van Kortryk[1], **T.L. Curtright**[2], **H. Alshal**[2,3]

[1] vankortryk@gmail.com

[2] Department of Physics, University of Miami, Coral Gables, Florida 33124, USA; curtright@miami.edu

[3] Department of Physics, Faculty of Science, Cairo University, Giza, 12613, Egypt; halshal@sci.cu.edu.eg

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Abstract. We fantasize about an alternative history for theories of electromagnetism and gravitation.

> "In the field of Astrobiology, the precise location, prevalence and age of potential extraterrestrial intelligence have not been explicitly explored." — Cai et al. [1]

1 Introduction

The Enceladians have developed an advanced society, an achievement quite remarkable for water-dwelling, bioluminescent creatures whose physical features are reminiscent of the Cephalopoda living at great depths in terrestrial seas. (But they live much longer than terrestrial cephalopoda – some as many as 500 Earth years!) Their scientific knowledge is impressive, although founded on experiments limited in spatial extent to the domain of their existence, an enclosed sea sheathed in a shell of ice many kilometers deep that surrounds their small Saturnian moon. They have no direct knowledge of anything outside this frozen crust, having never tried to pierce through it or even to probe its full thickness, mostly because of religious beliefs that have served them well for the past several millennia. Nevertheless, they have developed theories of electromagnetism, and of gravity.

They communicate mostly through images and light, particularly in one-on-one "conversations" by modifying the luminescent patterns on their skin, but also through the use of sound. Consequently, they understand very well the classical wave nature of both light and sound, as might be expected of highly intelligent beings with such ample life spans. Also, given their aquatic, ice-bound world, with the recurring deformations imposed on it by their unseen host planet, it is not surprising that they have long understood the mathematics of tensor calculus, especially in the context of continuum mechanics.

And so it happens, based on considerable thought and very careful experiments within the confines of their world, the Enceladians have ascertained that in almost all situations sound can be accounted for by a scalar field, ϕ, while electromagnetism can be explained using a vector field, ψ_μ, but to encode all its subtleties gravity is best described by a symmetric tensor, $\chi_{\mu\nu}$. On these points we are in agreement[1].

However, unlike us, they are not enamored with the concept of gauge invariance or the beauty of zero mass. They view zero mass as a peculiar limit, probably pathological, and the corresponding invariances as most likely just symptoms of some poorly understood illness for the underlying theory. Thus they describe classical electromagnetism using a Klein-Gordon field equation [2] with vector current sources, J_μ. Explicitly[2],

$$\left(\Box + m^2\right) \psi_\mu = J_\mu + \frac{1}{m^2}\, \partial_\mu \partial_\nu J^\nu \tag{1}$$

Here J_μ is most likely but not necessarily conserved, and for the Enceladians the value of m amounts to a question to be answered by experiment. So far their experiments only set limits on m and on the conservation of J_μ. Nonetheless, they understand that even if $\partial_\nu J^\nu \neq 0$, so long as $m \neq 0$ the divergence $\partial^\mu \psi_\mu$ does not correspond to any scalar radiation given off by the source.

Similarly, most Enceladian scientists describe gravity using the field equation

$$\begin{aligned}\left(\Box + M^2\right) \chi_{\mu\nu} = T_{\mu\nu} &+ \frac{1}{M^2}\left(\partial_\mu \partial^\rho T_{\rho\nu} + \partial_\nu \partial^\rho T_{\rho\mu} - \frac{1}{2}\, \eta_{\mu\nu} \partial^\rho \partial^\sigma T_{\rho\sigma}\right) \\ &+ \frac{2}{3M^4}\left(\partial_\mu \partial_\nu - \frac{1}{4}\, \eta_{\mu\nu} \Box\right) \partial^\rho \partial^\sigma T_{\rho\sigma}\,, \end{aligned} \tag{2}$$

where $T_{\mu\nu}$ is a symmetric, traceless (i.e. $0 \equiv \eta_{\mu\nu} T^{\mu\nu}$) tensor source, not necessarily conserved, and where the value of M is also to be determined by experiment. So far their experiments again only set limits on M and on the conservation of $T_{\mu\nu}$. And again, they are aware that even if $\partial^\rho T_{\rho\nu} \neq 0$, so long as $M \neq 0$ the divergence $\partial^\mu \chi_{\mu\nu}$ does not correspond to either vector or scalar radiation from localized $T_{\mu\nu}$'s, while the trace $\chi = \eta_{\mu\nu} \chi^{\mu\nu}$ is also not produced as scalar radiation from such sources. Indeed, directly from (2), $\left(\Box + M^2\right) \chi = 0$

[1] But of course, the Enceladians do *not* use Greek letters. We have transliterated their nomenclature.

[2] We take the liberty to use relativistic notation. (The Enceladians do not!) We use $\eta_{\mu\nu} = \mathrm{diag}\,(+1, -1, -1, -1) = \eta^{\mu\nu}$ to raise and lower indices, and sum over the same if repeated.

for traceless $T_{\mu\nu}$, so χ is a free field of mass M.[3]

Ongoing experimental studies [3] in their laboratories have convinced some Enceladians that the tensor $T_{\mu\nu}$ is not conserved, in general, but in fact its divergence can always be written as just the gradient of a scalar, S. Therefore, the full tensor source may be written in terms of a conserved tensor, $\Theta_{\mu\nu}$, plus this scalar part, $T_{\mu\nu} = \Theta_{\mu\nu} + \eta_{\mu\nu} S$. The scalar is then constrained by the traceless condition on $T_{\mu\nu}$ to be $S = -\Theta/4$, where $\Theta = \eta_{\mu\nu}\Theta^{\mu\nu}$. Thus,

$$T_{\mu\nu} = \Theta_{\mu\nu} - \frac{1}{4}\,\eta_{\mu\nu}\Theta\,, \quad \partial^\rho \Theta_{\rho\nu} = 0\,, \quad \partial^\rho T_{\rho\nu} = -\frac{1}{4}\,\partial_\nu \Theta\,. \tag{3}$$

So written, their gravitational field equation becomes

$$(\Box + M^2)\chi_{\mu\nu} = \Theta_{\mu\nu} - \frac{1}{4}\eta_{\mu\nu}\Theta - \frac{1}{6M^4}\Big(\partial_\mu\partial_\nu - \frac{1}{4}\eta_{\mu\nu}\Box\Big)(\Box + 3M^2)\Theta\,. \tag{4}$$

Consequently, not only is the trace χ a free field of mass M, but the divergence $\partial^\mu \chi_{\mu\nu}$ decouples as well, in the following sense:

$$(\Box + M^2)\Big[\partial^\mu \chi_{\mu\nu} + \frac{1}{8M^4}\left(\Box + 2M^2\right)\partial_\nu \Theta\Big] = 0\,. \tag{5}$$

That is to say, the combination within the brackets $[\cdots]$ is also a free field of mass M.

Recently, one of the more careful thinkers among Enceladian physicists (whose actual "name" is usually displayed bioluminescently on its skin, in a pattern most easily remembered by humans as the acronym "MGM") has realized a change of dependent variable that gives an alternative expression for their theory of gravity. Namely, after making the substitution

$$\chi_{\mu\nu} \to \chi_{\mu\nu} + \frac{1}{24M^4}\eta_{\mu\nu}(2M^2 + \Box)\Theta - \frac{1}{6M^4}\partial_\mu\partial_\nu\Theta \tag{6}$$

MGM obtained a simpler, indeed elegant, field equation

$$(\Box + M^2)\chi_{\mu\nu} = \Theta_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}\Theta - \frac{1}{3M^2}\,\partial_\mu\partial_\nu\Theta \tag{7}$$

and subsequently referred to it as a "teeter-totter" partial differential equation[4]: As the M^2 term on the LHS decreases, the $1/M^2$ term among the RHS sources

[3] The Enceladians are also aware that their method can be extended to higher spins without limit. For any J, with the higher spin field described by a totally symmetric rank J tensor, there are source terms involving as many as $2J$ derivatives of a traceless, symmetric, rank J source tensor on the RHS of the field equation, whose form is a straightforward generalization of (2). It is not necessary for that source tensor to be conserved.

[4] "Teeter-totter" is of course our translation of MGM's Enceladian description, and is not to be confused with the seesaw mechanism familiar to terrestrial physicists.

increases, and vice versa. In any case, it follows from (7) that the trace and the divergence of $\chi_{\mu\nu}$ decouple on a more equal footing[5] inasmuch as

$$(\Box + M^2)\left[\chi + \frac{1}{3M^2}\Theta\right] = 0 \tag{8}$$

$$(\Box + M^2)\left[\partial^\mu \chi_{\mu\nu} + \tfrac{1}{3M^2}\partial_\nu \Theta\right] = 0\,. \tag{9}$$

That is to say, the two combinations of fields and sources shown in (8) & (9) are both free fields of mass M.

After obtaining (7), MGM gave some consideration to its phenomenological consequences. Naively, from (7), the mass M cannot be too large, or the exponential suppression of static fields would be plainly evident even in experiments confined to the modest number of kilometers accessible to the Enceladians. Nor can M be too small if $\Theta \neq 0$, or else the $\partial_\mu \partial_\nu \Theta$ source on the RHS of (7) would be enhanced by the $1/M^2$ factor to the point of disagreement with various moderate precision, time-dependent experiments. Similar statements about m follow from (1) if $\partial_\nu J^\nu \neq 0$, but for theoretical as well as experimental reasons, the Enceladians have set strong limits on $\partial_\nu J^\nu$ and for most purposes treat J_μ as a conserved current. In that case, their experiments lead directly to upper limits on m.

Because of the size limitations imposed by their habitat and the environment in which they must work, direct experiments to set high precision limits on M are difficult but not impossible for the Enceladians. Looking for an exponential height variation in static gravitational fields is the best they can do, arriving at $M \leq 3 \times 10^{-47}$ kg corresponding to a distance scale of about ten kilometers. Similarly, electromagnetic wave dispersion experiments over distances of several kilometers present many challenges for them to infer a value for m, due mostly to background effects in their watery world. Consequently, their best limits come from electrostatic and magnetostatic experiments using meter-scale devices to find $m \leq 10^{-45}$ kg. (For comparison to terrestrial results, see [4] and [5].)

However, as MGM first emphasized, the limits on m and M are sufficiently different that they pose a potential problem. If the actual non-zero values of m and M are close to the Enceladian limits, then the ratio $m/M \approx 30$ which would imply the last source term in (7) is easily the most significant, and relatively quite large for many processes that can be measured with precision and without competing backgrounds, for example processes where light is effected by gravity. For smaller values of M, the problem becomes more pronounced, perhaps to the point of being in conflict with experiments on the scattering of light by light. In detail, the Enceladian expression of $\Theta_{\mu\nu}$ for electromagnetic fields in

[5] For the Enceladians "footing" should be understood in the muscular hydrostatic sense.

the absence of any J_μ sources is

$$\Theta_{\mu\nu} = -\kappa\Big[\partial_\mu\psi^\sigma\partial_\nu\psi_\sigma - \frac{1}{2}\eta_{\mu\nu}(\partial^\rho\psi^\sigma\partial_\rho\psi_\sigma - m^2\psi^\lambda\psi_\lambda) + \frac{1}{6}(\eta_{\mu\nu}\Box - \partial_\mu\partial_\nu)\psi^\lambda\psi_\lambda\Big], \quad (10)$$

where κ is a constant determined in static experiments involving masses on a torsion balance.[6] This tensor source is conserved, given (1) with $J_\mu = 0$, and has a non-zero trace

$$\partial^\mu\Theta_{\mu\nu} = -\kappa[\partial_\nu\psi_\sigma(\Box + m^2)\psi^\sigma] \doteq 0 \quad (11)$$

$$\Theta = -\kappa[m^2\psi^\lambda\psi_\lambda + \psi_\sigma(\Box + m^2)\psi^\sigma] \doteq -\kappa m^2\psi^\lambda\psi_\lambda\,. \quad (12)$$

Here "$\doteq$" means equality given $0 = \left(\Box + m^2\right)\psi^\sigma$. So the $1/M^2$ term in (7) is

$$\frac{\kappa m^2}{3M^2}\,\partial_\mu\partial_\nu(\psi^\lambda\psi_\lambda)\,. \quad (13)$$

Generically, this is 10^3 larger than the $\Theta_{\mu\nu}$ term on the RHS of (7), upon evaluating m/M right at the Enceladian limits given previously. This is experimentally untenable in many situations, even given the limitations of Enceladian science. MGM's conclusion is that whatever the direct experimental limits on M might be, if (13) is less than or comparable to experimentally constrained effects of the $\Theta_{\mu\nu}$ source, then m cannot be much larger than M.

But what if the Enceladian theory of gravity is flawed? Indeed, MGM has realized that a large m/M coefficient is probably a signal that additional contributions have been neglected in the source for $\chi_{\mu\nu}$. For large m/M most likely there are nonlinear terms involving the χ-field coupled to itself. MGM is grappling with nonlinear extensions of the theory, but has not yet managed to solve the problem. In that respect, MGM is considerably behind some mid-20th century terrestrial developments [6, 7].

Nevertheless, MGM's point about unusually large coupling of electromagnetic fields to gravity, for small but non-zero M, should be taken seriously by terrestrial physicists. The present value of m/M taken from the Particle Data Group summaries [4, 5], if evaluated right at the best accepted experimental limits, would give $m/M \approx 2 \times 10^{13}$?! So either m is actually much smaller than the current experimental limits, and/or M is much larger, or else higher-order nonlinear effects that have been omitted from the Enceladian model, so far, must be included to make it realistic.

Unknown to the Enceladians, even if the photon is absolutely massless, there is still an anomalous quantum trace for the electromagnetic energy-momentum tensor [8], namely, $\Theta = -\kappa N\hbar\alpha F^{\rho\sigma}F_{\rho\sigma}$ with $F_{\mu\nu}$ the usual electromagnetic field

[6] Indeed, within their experimental uncertainties the Enceladians do find $\kappa = 16\pi G/c^4$, i.e. $G = 6.67 \times 10^{-11}$ m^3 ,kg^{-1}s^{-2} after conversion to terrestrial SI units.

strength, α the fine-structure constant, and N a model-dependent number proportional to $\sum_j q_j^2 (-1)^{2s_j} (1 - 12 s_j^2)$ where the sum is over all physical particle states with helicities s_j and electric charges $q_j e$. In the Enceladian framework associated with field equation (7) this would lead to a source term enhanced by $1/M^2$ as given by

$$\frac{\kappa N \hbar \alpha}{M^2} \, \partial_\mu \partial_\nu \left(F^{\rho\sigma} F_{\rho\sigma} \right) \ . \tag{14}$$

For electric fields of frequency ω this term could give generic contributions to χ_{00} of order $\kappa\alpha\hbar^2\omega^2 E^2/M^2c^4$, thereby dominating the leading $\Theta_{\mu\nu}$ source contribution of order κE^2, if

$$Mc^2 \ll \hbar\omega\sqrt{\alpha} \, . \tag{15}$$

For frequencies as low as one MHz, this is troublesome since in that case $\hbar\omega\sqrt{\alpha}\big|_{\omega=2\pi\times 10^6} \approx 4 \times 10^{-10}$ eV, while the previously mentioned, rather crude Enceladian limit gives $Mc^2 < 2 \times 10^{-11}$ eV. This would give an enhancement of (14) compared to $\Theta_{\mu\nu}$ by two orders of magnitude! For higher ω the situation is worse. So here again, higher-order nonlinear effects that have been omitted from the Enceladian model, so far, would have to be included to make it realistic.

The situation brings to mind Vainshtein's proposal [9,10] that nonlinear gravitational effects can be exploited to avoid the van Dam-Veltman-Zakharov-Iwasaki discontinuity [11], where the latter was based on single particle exchange diagrams for massive versus massless gravitons. Only in that case, the disparity between experiments (planetary orbits compared to bending of light by the sun) caused by the discontinuity was merely a factor of $3/4 = O(1)$ whereas a disparity expressed by (15) could easily be many orders of magnitude.

The Enceladians had better get to work on those nonlinear terms!

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[7] "But even then I did have some thoughts about the future of the human race, especially in connection with the textbooks and the scientific romances of H.G. Wells. I loved to read his novels ... " — p 14, *The Quark and the Jaguar*

Appendix

A one-parameter field equation that leads to pure spin 2 radiation from a conserved source $\Theta_{\mu\nu}$ is given by

$$(\Box + M^2)\chi_{\mu\nu} = \Theta_{\mu\nu} + \frac{r-1}{4}\eta_{\mu\nu}\Theta + \frac{r}{3M^2}(\eta_{\mu\nu}\Box - \partial_\mu\partial_\nu)\Theta + \frac{1-r}{24M^4}(\eta_{\mu\nu}\Box - 4\partial_\mu\partial_\nu)(\Box + 3M^2)\Theta \quad \text{(A1)}$$

Choosing the parameter $r = 0$ or $r = 1$ gives simpler forms: The original O-P equation [6] for spin 2 is obtained for $r = 1$ while the Enceladian equation (4) is given by $r = 0$. Moreover, the substitution

$$\chi_{\mu\nu} \to \chi_{\mu\nu} + \frac{1}{24M^4}\eta_{\mu\nu}\big((2+6r)M^2 + (1-r)\Box\big)\Theta + \frac{1}{6M^4}(r-1)\partial_\mu\partial_\nu\Theta \quad \text{(A2)}$$

amounts to a field redefinition that leads to the equation

$$(\Box + M^2)\chi_{\mu\nu} = \Theta_{\mu\nu} - \frac{1}{24M^4}\big((8+6r)M^4 + 5M^2r\Box - r\Box^2\big)\eta_{\mu\nu}\Theta - \frac{1}{6M^4}\big((2+r)M^2 + r\Box\big)\partial_\mu\partial_\nu\Theta \quad \text{(A3)}$$

This greatly simplifies upon making the choice $r = 0$, to obtain the form found by MGM.

$$(\Box + M^2)\chi_{\mu\nu} = \Theta_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}\Theta - \frac{1}{3M^2}\partial_\mu\partial_\nu\Theta\,. \quad \text{(A4)}$$

It should be kept in mind that all choices for r lead to acceptable classical theories for the radiation of pure spin 2 fields of mass M. On the other hand, if the relative coefficients on the RHS of (A1) are modified, the spin 2 radiation may well be accompanied by spin 1 and/or spin 0 radiation [6,7]. In particular, to introduce spin zero radiation in the form of a propagating trace, χ, while retaining the decoupled divergence, in the sense of (9), it is only necessary to change both factors of $1/3$ on the RHS of (A4) to some other common value, say, $s/3$.

Dual formulations of massive gravity are also interesting, and may lead to additional understanding of the theory. For field equations this is easily done. To convert the symmetric tensor pure spin 2 $\chi_{\mu\nu}$ theory to a corresponding $\tau_{[\lambda\mu]\nu}$ tensor theory, or to convert a mixed spin 2 and spin 0 theory into a corresponding $\tau_{\lambda\mu\nu}$ tensor theory, where now $\tau_{\lambda\mu\nu} = \tau_{[\lambda\mu]\nu} \oplus \tau_{[\lambda\mu\nu]}$, it is sufficient to act with $\varepsilon^{\alpha\beta\gamma\delta}\partial^\lambda$ on both the field and the source terms in the $\chi_{\mu\nu}$ field equation, followed by contraction and symmetrization of the various indices. See [12,13].

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[2] For the Enceladians, m and M in the field equations are really inverse length scales, $1/\ell$ and $1/L$. We have expressed the equations in terms of masses through the familiar (to us) Compton wavelength relation, e.g. $L = \hbar/(Mc)$. But the Enceladians have not yet discovered $\hbar$, or anything else about quantum mechanics for that matter.

[3] Enceladian Journal of Physics, *to appear*.

[4] In addition to the PDG photon summary, which gives the limit $mc^2 < 1 \times 10^{-18}\ eV$, i.e. $m < 2 \times 10^{-54}\ kg$, the history of experiments to determine m is reviewed by Liang-Cheng Tu, Jun Luo, and George T. Gillies, "The mass of the photon" Rep. Prog. Phys. 68 (2005) 77–130. As noted by Tu et al., terrestrial scale experiments on the dispersion of radio waves in the 1930-40s gave $m \leq 5 \times 10^{-46}\ kg$. (See Table 1 for more recent experiments.) Maxwell's 1870s redo of Cavendish's experiment was not as good, by an order of magnitude, $m \leq 5 \times 10^{-45}\ kg$. (See Table 2 for more recent experiments.) Of course, all the current extra-terrestrial methods (e.g. as summarized in Table 3 of Tu et al.) are *not yet* available to the Enceladians.

[5] The PDG graviton summary gives the best experimental limit as $Mc^2 < 6 \times 10^{-32}\ eV$, i.e. $M \lessapprox 1 \times 10^{-67}\ kg$.

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