Modeling the Shape: Some Contemporary Approaches to Quadrupole-Octupole Deformations in Atomic Nuclei

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Abstract. We review some currently developing theoretical approaches to nuclear quadrupole-octupole (QO) deformations and dynamics. We depict the present status of the study of collective spectra within a model of axial QO vibrations and rotations, showing recent results of its full solution in the two-dimensional (2D) deformation space. Also, we illustrate the application of a deformed shell model (DSM) with BCS pairing to study the influence of QO deformations on the isomeric properties of heavy even-even nuclei.

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1 Introduction

The basic properties of atomic nuclei are determined by the nuclear shell structure [1]. In most nuclei it leads to the appearance of different kinds of shape deformation such as the QO deformation [2]. The latter causes collective phenomena such as alternating-parity bands (APBs) in even-even nuclei and quasi parity-doublet (QPD) spectra in odd-mass nuclei, with the observation of enhanced E1 and E3 transitions between levels with opposite parity. Also, the collective deformation modifies the intrinsic mean nucleonic field, balancing in its turn the shell structure and the attendant single-particle (s.p.) phenomena. A phenomenon deeply originating from the shell structure is the appearance of high-$K$ isomeric states (HKISs) [3], which similarly to the QO deformation are due to the coupling of appropriate s.p. orbitals near the Fermi level. The common physical roots of QO deformations and HKISs motivates their consideration in a common theoretical aspect, supposing a tight connection between them. In the present work we try to outline this aspect by reviewing in a consistent way the current progress in the study of nuclear APB and QPD spectra within a collective QO model approach [4]–[8] and the study of HKISs properties in dependence on QO deformations within a DSM+BCS approach [9–11].
In Section 2 we present the QO model and its full 2D solution in the axial QO variables. In Section 3 the DSM+BCS approach to HKISs is illustrated by emphasizing the role of the pairing strength on the predicted effects of QO deformation. In Section 4 we conclude by outlining the common theoretical aspects in both studies.

2 Model of Axial Quadrupole-Octupole Vibrations and Rotations

The collective motion associated with QO shapes is considered to be a QO vibration on top of which rotation modes are built. In the best known QO-deformation representatives, Rn, Ra and Th isotopes, the APB forms a single octupole band at higher angular momenta, indicating the appearance of a quite stiff octupole deformation. This structure is related to oscillations of the system with respect to one-dimensional (1D) double-well octupole potential [12]. In much more other nuclei, both even-even and odd-mass ones, in the rare-earth, actinide and heavier mass regions, the APBs and QPDs do not form single-band sequences, but rather indicate the presence of a soft QO mode.

A common model framework for description of QO deformations with different degrees of softness has been developed in [4]–[8]. The case of stiff QO deformations was described by the so-called QO Rotation Model (QORM) combined with octupole vibrations in a 1D double-well potential with angular-momentum dependent barrier [4]. For the case of soft QO deformations the latter was generalized to a 2D QO potential in the space of axial quadrupole and octupole variables with embedded rotation term providing an infinite angular-momentum dependent core at the zero-deformation point [5]. Though looking simple the treatment of the general 2D eigenvalue problem appears to be far from trivial. In [5] it was shown that if certain relations between the quadrupole and octupole mass, stiffness and inertia parameters are imposed the 2D problem can be reduced to a 1D one for which a simple analytic solution can be obtained. The particularly assumed relations correspond to the imposition of a common, coherent oscillation frequency for the quadrupole and octupole vibration modes. It appeared that the analytic model formalism based on this coherent QO mode (CQOM) quite reasonably describes the structure of yrast and non-yrast APBs and QPDs and the attendant B(E1)–B(E3) transition probabilities in wide ranges of even-even and odd-mass nuclei [5–8]. Nevertheless, the imposed coherent constraints lead to certain limitations in the geometric interpretation of the physical pattern, such as the ultimately appearing ellipsoidal bottom of the QO deformation potential as well as some restrictions on the model descriptions due to the imposed purity of the coherent QO modes. To overcome these limitations one has to solve the complete unrestricted 2D problem for the coupled QO vibrations and rotations. Here we show how this can be done by using the analytic solution of the constrained CQOM as a basis.
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The general QO model Hamiltonian $H_{qo} = T_{qo} + U_{qo}$ is given by [5, 6]

$$
T_{qo} = -\frac{\hbar^2}{2B_2} \frac{\partial^2}{\partial \beta_2^2} - \frac{\hbar^2}{2B_3} \frac{\partial^2}{\partial \beta_3^2};
$$

$$
U_{qo} = \frac{1}{2} C_2 \beta_2^2 + \frac{1}{2} C_3 \beta_3^2 + \frac{X(I, K)}{d_2 \beta_2^2 + d_3 \beta_3^2},
$$

where $\beta_2$ and $\beta_3$ are axial quadrupole and octupole variables, respectively, $B_2$ ($B_3$), $C_2$ ($C_3$) and $d_2$ ($d_3$) are quadrupole (octupole) mass, stiffness and inertia parameters, respectively and

$$
X(I, K) = \frac{1}{2} \left[ d_0 + I(I + 1) - K^2 + \pi a \delta_{K, \frac{1}{2}} (-1)^{I+1/2} \left( I + \frac{1}{2} \right) \right].
$$

The parameter $d_0$ determines the potential shape at $I = 0$ and $\pi$ is the total parity of the state. In odd-mass nuclei the Coriolis decoupling factor $a$ was originally taken as an adjustable parameter [6], whereas in a later work it was calculated by using DSM+BCS within a parity-projection particle-core coupling scheme [13]. The shapes of the potential $U_{qo}$ are illustrated in Figures 1 and 2 in Ref. [5], where in Figure 2 it is shown that under the constraint $C_2/d_2 = C_3/d_3$ it possesses an ellipsoidal bottom. This implies the convenience of introducing ellipsoidal coordinates such that $\beta_2 = p \eta \cos \phi$, $\beta_3 = q \eta \sin \phi$, with $p = \sqrt{d/d_2}$, $q = \sqrt{d/d_3}$ and $d = (d_2 + d_3)/2$. Then the kinetic term $T_{qo}(\eta, \phi)$ of the Hamiltonian appears in the form present in Eq. (11) of [5], whereas the potential term becomes

$$
U_{qo}(\eta, \phi) = \frac{C_2 d \eta^2 \cos^2 \phi}{2d_2} + \frac{C_3 d \eta^2 \sin^2 \phi}{2d_3} + \frac{X(I)}{d \eta^2}.
$$

Further, under the assumption of coherent QO oscillations with a frequency $\omega = \sqrt{C_2/B_2} = \sqrt{C_3/B_3} \equiv \sqrt{C/B}$ the spectrum is obtained in the form [5, 6]

$$
E_{nk}(I, K) = \hbar \omega \left[ 2n + 1 + \sqrt{k^2 + bX(I, K)} \right], \quad n = 0, 1, 2, \ldots; \quad k = 1, 2, 3, \ldots,
$$

where $b = 2B/(\hbar^2 d)$. The QO vibration wave function is

$$
\Phi_{nk}^{\pm}(\eta, \phi) = \psi_{nk}^{I}(\eta) \varphi_{k}^{\pm}(\phi),
$$

where the “radial” part $\psi_{nk}^{I}(\eta)$ involves generalized Laguerre polynomials in the variable $\eta$ [5]. The “angular” part in the variable $\phi$ is obtained under the boundary condition $\varphi(-\pi/2) = \varphi(\pi/2) = 0$ which effectively reduces the solution to $\beta_2 > 0$ and determines the parity, $\pi_\varphi = \pm$, of the wave function

$$
\varphi_{k}^{+}(\phi) = \sqrt{2/\pi} \cos(k\phi), \quad k = 1, 3, 5, \ldots \quad (\pi_\varphi = +),
$$

$$
\varphi_{k}^{-}(\phi) = \sqrt{2/\pi} \sin(k\phi), \quad k = 2, 4, 6, \ldots \quad (\pi_\varphi = -).
$$
Then the total wave function for APB states in even-even nuclei has the form [5]

\[ \Psi_{nIM0}(\eta, \phi) = \sqrt{\frac{2I + 1}{8\pi^2}} D_{M0}^I(\theta) \psi_n^\pi(\eta) \varphi^\pi(\phi), \] (8)

where \( D_{M0}^I(\theta) \) is the Wigner function. In odd-mass nuclei the total core+particle wave function for the QPD states is given by [6]

\[ \Psi_{nkIMK}(\eta, \phi) = \sqrt{\frac{2I + 1}{16\pi^2}} |\Phi_{nkI}(\eta, \phi)| \left[ D_{MK}^I(\theta) \mathcal{F}_K \right. \\
+ \left. \pi \cdot \pi_{sp} (-1)^I K D_{M-K}^I(\theta) \mathcal{F}_{-K} \right], \]

where \( \mathcal{F}_K \) and \( \pi_{sp} \) are the wave function and parity of the s.p. state, respectively.

The structure of the spectrum in CQOM is determined in (4) by the quantum numbers \( n \) and \( k \), with the latter being responsible for the parity \( \pi_\phi \) and the related parity-shift effects. In odd-mass nuclei one has the relation \( \pi = \pi_\phi \cdot \pi_{sp} \), which determines the core parity \( \pi_\phi = \pi \cdot \pi_{sp} \). Thus, the wave function (9) is completely determined by the observed (total) parity \( \pi \) of the state and the assumed parity of the odd nucleon, \( \pi_{sp} \). (For details see [5, 6].) By using the CQOM formalism the yrast and non-yrast APBs and QPBs and the attendant \( B(E1), B(E2) \) and \( B(E3) \) transition probabilities in various even-even and odd-mass nuclei were described [7, 8].

Now, to obtain the spectrum of the general Hamiltonian (1) with arbitrary values of the mass, stiffness and inertia parameters it appears very convenient to diagonalize it by using as a basis the analytic CQOM wave function (5). For this reason we take (1) in ellipsoidal variables, Eq. (11) in [5] and Eq. (3) above, and consider its matrix elements between CQOM states with given \( I \) [14]

\[ \langle n' k' | H_{wp} | n k \rangle = \frac{d}{\sqrt{d_2 d_3}} \int_{-\frac{Z}{2}}^{\frac{Z}{2}} \int_0^\infty \psi_{n' k'}^I(\eta) \varphi_{k'}(\phi) H_{wp}(\eta, \phi) \psi_{nk}^I(\eta) \varphi_k(\phi) \eta d\eta d\phi, \] (10)

where \( |n k\rangle \equiv \Phi_{nkI}^\pm(\eta, \phi) \). For the kinetic matrix element \( \langle n' k' | T_{qo} | n k \rangle \) we obtain a lengthy, but well determined expression in terms of products of matrix elements of powers of \( \eta, \sin \phi \) and \( \cos \phi \). Much simpler expression for the potential matrix element \( \langle n' k' | U_{qo} | n k \rangle \) directly comes from Eq. (3). Thus we get the matrix element (10) in a form involving integrals of products of powers of \( \sin \phi \) and \( \cos \phi \) and integrals of the powers of \( \eta \). The former are simplified and calculated with Mathematica while the later are easily calculated by using a known analytic expression for integrals involving two Laguerre polynomials [15]

\[ \int_0^\infty t^{\alpha-1} e^{-pt} L_\alpha^\lambda(p t) L_\beta^\beta(p t) dt = \frac{p^{-\alpha} \Gamma(\alpha) \Gamma(n-\alpha+\beta+1) \Gamma(m+\lambda+1)}{m! n! \Gamma(1-\alpha+\beta) \Gamma(\lambda+1)} \times 3F_2(-m, \alpha - \beta; -n + \alpha - \beta, \lambda + 1; 1), \] (11)
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where $3F_2$ is the generalized hypergeometric function. The use of the CQOM analytic basis has the following advantages: i) easily calculated analytic matrix elements; ii) built-in boundary condition and parity properties of the wave functions; iii) no singularity at the zero deformation point.

To get the spectrum the Hamiltonian is diagonalized for each angular momentum $I$ in a set of CQOM functions limited to large enough $n$ and $k$ oscillator-number values. The yrast APB or QPD band is determined by taking for each angular momentum $I$ the lowest eigenvalue the eigenfunction of which has the relevant parity. Each wave function is obtained in the form of an expansion

$$\tilde{\Psi}_I(\eta, \phi) = \sum_{n,k} C_{nkI} \Psi_{nkI}(\eta, \phi),$$

(12)

where the coefficients $C_{nkI}$ are determined. Hence the matrix elements of the electric transition operators are obtained in the form of sums over $n$ and $k$ of CQOM matrix elements. Thus all reduced transition probabilities are calculated by using the formalism elaborated in the CQOM approach [7].

Here we illustrate the above approach in the yrast APB of the nucleus $^{152}$Sm for which CQOM descriptions where obtained in [5] and [7] (including non-yrast APBs). The unconstrained model parameters were adjusted to experimental levels [16] and B(E1)-B(E3) transition probabilities [17, 18] with the values $B_2 = 109.0\hbar^2/\text{MeV}$, $B_3 = 100.08\hbar^2/\text{MeV}$, $C_2 = 16.79\text{MeV}$, $C_3 = 339.8\text{MeV}$, $d_2 = 120.7\hbar^2/\text{MeV}$, $d_3 = 4744\hbar^2/\text{MeV}$, $d_0 = 81.40\hbar^2$ and $e_1^{\text{eff}} = 2.85e$ (effective charge, see [7] for explanation). In Table 1 the levels obtained in the

<table>
<thead>
<tr>
<th>$I^\pi$</th>
<th>CQOM rms=49 keV</th>
<th>2DQOM rms=33 keV</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^-$</td>
<td>853.73</td>
<td>889.20</td>
<td>963.354</td>
</tr>
<tr>
<td>$2^+$</td>
<td>112.23</td>
<td>109.00</td>
<td>121.782</td>
</tr>
<tr>
<td>$3^-$</td>
<td>994.59</td>
<td>1023.22</td>
<td>1041.114</td>
</tr>
<tr>
<td>$4^+$</td>
<td>357.52</td>
<td>350.25</td>
<td>366.479</td>
</tr>
<tr>
<td>$5^-$</td>
<td>1235.09</td>
<td>1252.12</td>
<td>1221.48</td>
</tr>
<tr>
<td>$6^+$</td>
<td>706.33</td>
<td>699.17</td>
<td>706.88</td>
</tr>
<tr>
<td>$7^-$</td>
<td>1557.94</td>
<td>1559.44</td>
<td>1505.61</td>
</tr>
<tr>
<td>$8^+$</td>
<td>1129.65</td>
<td>1129.51</td>
<td>1125.35</td>
</tr>
<tr>
<td>$9^-$</td>
<td>1945.59</td>
<td>1928.33</td>
<td>1879.11</td>
</tr>
<tr>
<td>$10^+$</td>
<td>1604.99</td>
<td>1618.84</td>
<td>1609.23</td>
</tr>
<tr>
<td>$11^-$</td>
<td>2382.93</td>
<td>2344.09</td>
<td>2326.96</td>
</tr>
<tr>
<td>$12^+$</td>
<td>2116.62</td>
<td>2149.75</td>
<td>2148.51</td>
</tr>
<tr>
<td>$13^-$</td>
<td>2857.99</td>
<td>2794.89</td>
<td>2833.25</td>
</tr>
<tr>
<td>$14^+$</td>
<td>2653.93</td>
<td>2708.84</td>
<td>2736.01</td>
</tr>
</tbody>
</table>

Table 1. Theoretical 2DQOM and CQOM yrast APB levels (in keV) of $^{152}$Sm compared to experimental data [16].
Table 2. Theoretical (2DQOM) and experimental [17,18] reduced transition probabilities for $^{152}\text{Sm}$.

<table>
<thead>
<tr>
<th>Mult</th>
<th>Transition</th>
<th>2DQOM [W.u.]</th>
<th>Exp [W.u.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>E2</td>
<td>$2^+ \rightarrow 0^+$</td>
<td>139</td>
<td>144(3)</td>
</tr>
<tr>
<td>E2</td>
<td>$4^+ \rightarrow 2^+$</td>
<td>209</td>
<td>209(3)</td>
</tr>
<tr>
<td>E2</td>
<td>$6^+ \rightarrow 4^+$</td>
<td>251</td>
<td>245(5)</td>
</tr>
<tr>
<td>E2</td>
<td>$8^+ \rightarrow 6^+$</td>
<td>286</td>
<td>285(14)</td>
</tr>
<tr>
<td>E2</td>
<td>$10^+ \rightarrow 8^+$</td>
<td>316</td>
<td>320(3)</td>
</tr>
<tr>
<td>E1</td>
<td>$1^- \rightarrow 0^+$</td>
<td>0.0041</td>
<td>0.0042(4)</td>
</tr>
<tr>
<td>E1</td>
<td>$1^- \rightarrow 2^+$</td>
<td>0.0085</td>
<td>0.0077(7)</td>
</tr>
<tr>
<td>E2</td>
<td>$3^- \rightarrow 2^+$</td>
<td>0.0063</td>
<td>0.0081(16)</td>
</tr>
<tr>
<td>E2</td>
<td>$3^- \rightarrow 4^+$</td>
<td>0.0089</td>
<td>0.0082(16)</td>
</tr>
<tr>
<td>E3</td>
<td>$3^- \rightarrow 0^+$</td>
<td>8.87</td>
<td>14(2)</td>
</tr>
</tbody>
</table>

present 2D QO model (2DQOM) are compared with those in CQOM [5] and with the experimental levels [16]. The theoretical and experimental transition probabilities are compared in Table 2. We see that the fits of the unconstrained parameters essentially improve the APB description compared to CQOM, quality being measured by the standard root mean square (rms) value. Also the good description of transition rates is seen.

In Figure 1 the QO potential shapes corresponding to the model parameters for $^{152}\text{Sm}$ are plotted. The left plot represents the full potential shape, whereas the right one illustrates the corresponding effective potential for $\beta_2 > 0$ with the built-in infinite wall at $\beta_2 = 0$. We see that now the full 2DQOM solution suggests a QO potential the bottom of which is not ellipsoidal but possesses single quadrupole and octupole minima. The right plot visualizes the effective oscillation of the nuclear shape between the positive and negative $\beta_3$ minima with a motion around the zero-deformation core and simultaneous tunneling through a 2D potential barrier. At the same time the obtained potential shape provides

Figure 1. (Color online) 2DQOM model shapes for the full (left) and the effective ($\beta_2 > 0$) (right) QO potential obtained for $^{152}\text{Sm}$.
model estimates for the quadrupole deformation $\beta_2 \sim 0.37$ (somewhat overestimated) and for the octupole deformation $\beta_3 \sim 0.09$ (quite reasonable) in $^{152}\text{Sm}$.

3 Octupole Deformations in High-$K$ Isomeric States

Recently it was shown within a DSM+BCS approach, that the properties of some HKISs in heavy even-even nuclei and especially their magnetic dipole moments exhibit pronounced sensitivity to the octupole deformation [9–11]. In a number of nuclei, in the regions of rare-earth (Nd, Sm and Gd), actinide (U, Pu and Cm), heavier (Fm and No) and superheavy ($^{270}\text{Ds}$) elements, minima in the neutron two-quasiparticle (2qp) energy surfaces were indicated at non-zero octupole deformation. Groups of nuclei with pronounced and with shallow 2qp energy minima were outlined [11] suggesting the possible presence of octupole deformation or at least octupole softness in the corresponding HKISs. This finding shows the need of further more detailed analysis of the mechanism which causes the appearance of 2qp energy minima as well as the factors which determine their evolution in deformation space. Therefore, in this section we examine the effect of the blocking of the excited 2qp configuration as well as the role of the pairing strength for the appearance, the depth and the position of the 2qp energy minimum in the QO deformation space.

In brief, the approach involves calculation of 2qp energy and magnetic moment over a net in the QO deformation parameters space ($\beta_2, \beta_3$). For each isomeric state/nucleus the calculation provides a 2qp-energy surface in ($\beta_2, \beta_3$) and a 2D pattern for the magnetic dipole moment in the isomeric state as a function of $\beta_2$ and $\beta_3$. In these calculations the BCS pairing constants for neutrons(n)/protons(p) are taken as $G_{n/p} = (g_0 \mp g_1 \frac{N-Z}{A}) / A$. The parameters $g_0$ and $g_1$ are originally taken in [19] as $g_0 = 19.2$ MeV and $g_1 = 7.4$ MeV. In the DSM+BCS calculations performed in [9–11] without blocking the excited orbitals the parameter $g_0$ was slightly decreased to $g_0 = 17.8$ MeV, to provide for the different deformations overall gap values comparable with the experimentally estimated gaps in the considered nuclei. To examine the influence of the pairing strength on the HKIS we vary the parameter $g_0$ or, more precisely, we perform calculations for a given 2qp energy pattern with several different values of this parameter. Details about the used DSM+BCS approach and the calculation procedures can be seen e.g. in [9].

Here, first we illustrate results of calculations in which the orbitals from which the isomeric state is formed are not blocked in the BCS procedure. In Figure 2 the result for the $K^\pi = 8^-$ isomeric state based on the neutron $(\nu)$ \{\nu7/2[624] \otimes \nu9/2[734]\} configuration in $^{254}\text{No}$ is given as one of the best examples for the influence of the octupole deformation. The 2qp energy surface in Figure 2 (left) shows the presence of a considerably deep minimum, about 0.32 MeV, at non-zero octupole deformation ($\beta_2 = 0.302, \beta_3 = 0.212$). The obtain-
Figure 2. (Color online) Two-quasiparticle energy and magnetic moment of the $K^\pi = 8^- \{\nu 7/2[624] \otimes \nu 9/2[734]\}$ configuration in $^{254}$No calculated within DSM+BCS without blocking as functions of $\beta_2$ and $\beta_3$.

ing of such a minimum suggests the possibility for stable octupole deformation in this state. The plot in Figure 2 (right) shows that the magnetic moment in the $K^\pi = 8^-$ isomer of $^{254}$No essentially changes in the direction of non-zero octupole deformation, whereas its value at $\beta_3 = 0$ shows a very weak dependence on the quadrupole deformation. The appearance of the 2qp energy minimum in Figure 2 (left) as well as the behaviour of the magnetic moment in Figure 2 (right) can be explained in relation to the crossing of the neutron $7/2[624]$ and $9/2[734]$ orbitals at some non-zero octupole deformation similarly to the case of $K^\pi = 8^-$ isomer in $^{244}$Pu (see Figure 1 in [9]).

Now, let us consider the case of DSM+BCS calculations in which the two orbitals providing the HKIS are blocked in the solution of the BCS gap equation. Initially the calculation was applied to the $K^\pi = 8^-$ isomer in $^{254}$No with Nilsson’s original parameter values $g_0 = 19.2$ MeV and $g_1 = 7.4$ MeV [19] used in the paring constants $G_{n/p}$. The analysis of the results showed that in most parts of the ($\beta_2, \beta_3$) deformation space the gap equation does not possess a solution and the 2qp energy surface can not be obtained with a relevant shape allowing us to make any conclusion. For the previously used value of the parameter $g_0 = 17.8$ MeV [9–11] the problem with the BCS solution becomes even stronger. Then we performed calculations with slightly larger values of the parameter $g_0 > 19$ MeV by keeping $g_1 = 7.4$ MeV. The results of calculations with several $g_0$ values, $g_0 = 20, 21, 22$ and 23 MeV, are given in Figure 3. It is immediately seen that both the positions and the depths of the 2qp minima in the deformation space depend on the value of the pairing parameter $g_0$. Thus for $g_0 = 20$ MeV the minimum is positioned at $(\beta_2 = 0.25, \beta_3 = 0.07)$ with a relative depth about 0.05 MeV; for $g_0 = 21$ MeV the position of the minimum is at $(\beta_2 = 0.25, \beta_3 = 0.10)$ with a relative depth about 0.08 MeV; for $g_0 = 22$ MeV the minimum is at $(\beta_2 = 0.26, \beta_3 = 0.13)$ with a relative depth about 0.10 MeV. We note the relatively shallow 2qp minima appearing for this nucleus when the blocking effect is taken into account.
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Figure 3. (Color online) Two-quasiparticle energy of the $K^\pi = 8^- \{\nu7/2[624] \otimes \nu9/2[734]\}$ configuration in $^{254}$No as a function of $\beta_2$ and $\beta_3$ calculated by DSM+BCS with blocking for different values of the pairing parameter $g_0$.

4 Conclusion

We reviewed series of studies illustrated with particular recent results in the subjects of nuclear collective spectra and HKISs from the common aspect of the QO shape-deformation phenomenon. The consideration depicts the common structural roots of the underlying collective and s.p. dynamics. In both cases it was demonstrated that the model assumption of complex shape deformation either in the collective or in the s.p. Hamiltonian leads to relevant estimates for the respective collective (spectra and transition probabilities) or s.p. (qp energies and magnetic moments) properties in dependence on the particular deformation parameters (or variables). Moreover, we show that in both considered approaches the requirements of physical relevance leads to model estimates for the most appropriate shape deformation. Thus, the diagonalization of the full 2DQO Hamiltonian illustrated for $^{152}$Sm provides unambiguous model estimates for the QO potential and the related QO deformation modes. In a similar way the requirement for a minimum in the 2qp energy obtained by DSM+BCS calculations, illustrated for $^{254}$No, provides estimates for the QO deformations which may favour the forming of a HKIS. These results show the relevance of
the currently developing collective and microscopic approaches to nuclear QO deformations and encourage their further development and application to study nuclear complex-shape dynamics.

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