Bulg. J. Phys. **41** (2014) 274–290

Magnetized Cosmological Models in Saez Ballester Theory of Gravitation

S.D. Katore[1], **A.Y. Shaikh**[2]

[1]Department of Mathematics, S.G.B. Amravati University, Amravati-444602, India
[2]Department of Mathematics, Dr.B.N. College of Engineering & Technology, Yavatmal-445001, India

Received *14 November 2014*

Abstract. Bianchi type-I magnetized cosmological model in scalar tensor theory proposed by Saez and Ballester (*Phys. Lett. A* **113** (1986) 467) with perfect fluid as a source is investigated. The source of the magnetic field is due to an electric current produced along the x-axis. F_{23} is the non-vanishing component of electromagnetic field tensor. To get deterministic model, it has been assumed that the component σ_1^1, of eigenvalue of shear tensor σ_j^i is proportional to expansion scalar θ. The behavior of models in presence and absence of magnetic field with physical properties are discussed.

PACS codes: 04.20.-q, 98.80.Cq

1 Introduction

In the last few decades there has been much interest in alternative theories of gravitation, especially the scalar tensor theories proposed by Brans and Dicke [1], Nordvedt [2], Barber [3], Saez and Ballester [4], Lau and Prokhovnik [5] etc. Brans and Dicke [1] scalar-tensor theory of gravitation introduces an additional scalar field φ beside the metric tensor g_{ij} and a dimensionless value coupling constant ω. This theory tends to general relativity for large value of the coupling constant ($\omega > 500$). In Saez and Ballester theory [4], the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields in which an anti-gravity regime appears despite the dimensionless behavior of the scalar field. This theory suggests a possible way to solve the missing matter problem in non-at FRW models.

The field equations in the scalar tensor theory proposed by Saez and Ballester [4] are

$$G_{ij} - \omega\varphi^n \left(\varphi_{;i}\varphi_{;j} - \frac{1}{2}g_{ij}\varphi_{,k}\varphi^{,k}\right) = -T_{ij} \quad , \tag{1}$$

and the scalar field φ satisfies the equation

$$2\varphi^n \varphi^i_{;i} + n\varphi^{n-1}\varphi_{,k}\varphi^{,k} = 0 \quad , \tag{2}$$

where G_{ij} is the Einstein tensor, T_{ij} is the stress tensor of matter, n is an arbitrary constant, ω is a dimensionless coupling constant and other symbols have their usual meaning. Here comma and semicolon denote partial and covariant differentiation respectively.

Also the energy conservation equation

$$T^{ij}_{;j} = 0 \tag{3}$$

is a consequence of the field equations.

Saez and Ballester [4] assumed the Lagrangian

$$L = R - \omega\varphi^n(\varphi_{,\alpha}\varphi^{,\alpha}), \tag{4}$$

where R is the scalar curvature, φ is the dimensionless scalar field, ω and η are arbitrary dimensionless constants and $\varphi^{,\alpha} = \varphi_{,\alpha}g^{\gamma\alpha}$. For scalar field having the dimension φG^{-1}, the Lagrangian given by equation (4) has different dimensions. However it is a suitable Lagrangian in the case of a dimensionless scalar field.

From the above Lagrangian one can build the action

$$I = \int_{\Sigma} (L + GL_m)(-g)^{\frac{1}{2}}\, dxdydzdt, \tag{5}$$

where L_m is the matter Lagrangian, $g = |g_{ij}|$, Σ is an arbitrary region of integration and $G = -8\bar{\Lambda}$. By considering arbitrary independent variations of the metric and the scalar field vanishing at the boundary of Σ, the variational principle

$$\delta I = 0 \tag{6}$$

leads to the Saez-Ballester field equations (1),(2) and (3).

In earlier literature, cosmological models within the framework of Sáez-Ballester scalar-tensor theory of gravitation, have been studied by Singh and Agrawal [6,7], Ram and Tiwari [8], Singh and Ram [9]. Mohanty and Sahu [10,11] have studied Bianchi type-VI_0 and Bianchi type-I models in Saez-Ballester theory. In recent years, Tripathi et al. [12], Reddy et al. [13,14], Reddy and Naidu [15], Rao et al. [16-18], Adhav et al. [19], Katore et al. [20], Sahu [21], Singh [22], Pradhan and Singh [23], Socorro and Sabido [24] and Jamil et al. [25] have obtained the solutions in Sáez-Ballester scalar-tensor theory of gravitation in different context. Recently, Naidu et al. [26,27] and Reddy et al. [28] have studied LRS Bianchi type-II models in Sáez and Ballester scalar tensor theory of gravitation in different context.

One of the well-established fact today is the occurrence of magnetic fields on galactic scale and their importance for a variety of astrophysical phenomena is generally acknowledged [29-33]. The occurrence of magnetic fields on a galactic scale is a well-established fact today,and its importance for a variety of astrophysical phenomena is generally acknowledged as pointed out by Zeldovich et al. [29]. Also Harrison [30] suggests that magnetic field could have a cosmological origin. As natural consequences, we should include magnetic fields in the energy-momentum tensor of the early universe. It is reasonable to consider magnetic fields in the energy-momentum tensor of the early universe. The magnetized string cosmological models are studied by some cosmologists. Banerjee et al [34], Chakraborty [35], Tikekar et al [36,37], Bali et al.[38-43], Yadav et al. [44,45], Pradhan et al [46-49] have investigated Bianchi type I, II, III, V, IX, VI_0 and cylindrically symmetric magnetized cosmological models in presence and absence of bulk viscosity and cosmic strings. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than Robertson-Walker model [50].

With these motivations and following the technique of Bali and Chandani [51], in this paper, we have investigated Bianchi type-I magnetized perfect fluid model in scalar tensor theory proposed by Saez-Ballester. To get deterministic model of the universe, we have assumed the component σ_1^1 of shear σ_j^i is proportional to expansion scalar θ.The behaviors and physical properties of the model in presence and absence of magnetic field are discussed.

2 Metric and Field Equations

We consider the Bianchi type-I metric in the form.

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \quad , \tag{7}$$

where A, B, C are functions of t alone.

A new class of exact solutions of Einstein's modified field equations in inhomogeneous space-time for perfect fluid distribution with variable magnetic permeability and time dependent gauge function β within the framework of Lyra geometry is investigated by Singh and Singh [52]. Bianchi type-I cosmological model for barotropic fluid distribution with magnetic field in Lyra geometry is investigated by Bali and Vadhwani [53]. Bali and Chandnani [51] have investigated Bianchi type-I cosmological model with time dependent gauge function β for perfect fluid distribution within the framework of Lyra geometry. Bianchi type-I string dust cosmological models in the presence and absence of magnetic field in the framework of Lyra geometry is investigated by Bali et al. [54].

The energy momentum for perfect fluid distribution in the presence of magnetic field is given by

$$T_j^i = (\rho + p)\, u_i u^j + p g_j^i + E_j^i \quad , \tag{8}$$

where ρ is the energy density, p is the isotropic pressure, and v_i and x_i satisfy the condition

$$v_i v^i = -x_i x^i = -1 \tag{9}$$

and

$$x_i v^i = 0\ . \tag{10}$$

E_j^i is the electromagnetic field given as Lichnerowicz [55]

$$E_i^j = \overline{\mu} \left\{ |h|^2 \left(v_i v^j + \frac{1}{2} g_i^j \right) - h_i h^j \right\}. \tag{11}$$

In the above v_i is the flow vector satisfying

$$g_{ij} v^i v^j = -1 \tag{12}$$

and $\overline{\mu}$ is the magnetic permeability and h_i the magnitude flux vector defined by

$$h_i = \frac{\sqrt{-g}}{2\overline{\mu}} \in_{ijkl} F^{kl} v^i \quad , \tag{13}$$

where F_{kl} is the electromagnetic field tensor and $\in_{ijkl}$ is the Levi-Civita tensor density.

We assume that coordinates to be co-moving so that

$$v^1 = v^2 = v^3 = 0 \qquad \text{and} \qquad v^4 = 1, \qquad \text{i.e.} \qquad v^i = (0, 0, 0, 1)\,. \tag{14}$$

We assume that current is flowing along the x-axis, so the magnetic field is in the yz plane. Thus $h_1 \neq 0$, $h_2 = h_3 = h_4 = 0$ and F_{23} is the only non-vanishing component of F_{ij}. This leads to $F_{12} = F_{13} = 0$ by virtue of equation (13). We also find $F_{14} = F_{24} = F_{34} = 0$ due to the assumption of infinite electrical conductivity of the fluid. [(Roy Martens [56]]. A cosmological model, which contains a global magnetic field, is necessarily anisotropic since the magnetic vector specifies a preferred spatial direction [57].

The Maxwell equations

$$F_{ij;k} + F_{jk;i} + F_{ik;j} = 0, \tag{15}$$

and $F^{ij}_{;j} = 0$, i.e.

$$\frac{\partial}{\partial x^j} \left(F^{ij} \sqrt{-g} \right) = 0\ , \tag{16}$$

are satisfied by $F_{23} = H =$ const. (say).

Equation (13) leads to

$$h_1 = \frac{AH}{\overline{\mu} BC}. \tag{17}$$

Since $|h|^2 = h_l h^l = h_1 h^1 = g^{11}(h_1^2)$, therefore

$$|h|^2 = \frac{H^2}{\overline{\mu} B^2 C^2}. \tag{18}$$

Using equations (17) and (18) in (11), we have

$$E_1^1 = \frac{-H^2}{2\overline{\mu} B^2 C^2} = -E_2^2 = -E_3^3 = E_4^4. \tag{19}$$

Using equations (19) and (8), we have

$$\begin{aligned} T_1^1 &= p - \frac{H^2}{2\overline{\mu} B^2 C^2}, \qquad T_2^2 = p + \frac{H^2}{2\overline{\mu} B^2 C^2}, \\ T_3^3 &= p + \frac{H^2}{2\overline{\mu} B^2 C^2}, \qquad T_4^4 = -\rho - \frac{H^2}{2\overline{\mu} B^2 C^2}. \end{aligned} \tag{20}$$

The modified Einstein's field equations (1) and (2) for metric (7) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{w\varphi^n \varphi_4^2}{2} = -\left(p - \frac{H^2}{2\overline{\mu} B^2 C^2}\right), \tag{21}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{w\varphi^n \varphi_4^2}{2} = -\left(p + \frac{H^2}{2\overline{\mu} B^2 C^2}\right), \tag{22}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{w\varphi^n \varphi_4^2}{2} = -\left(p + \frac{H^2}{2\overline{\mu} B^2 C^2}\right), \tag{23}$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} + \frac{w\varphi^n \varphi_4^2}{2} = \rho + \frac{H^2}{2\overline{\mu} B^2 C^2}, \tag{24}$$

$$\varphi_{44} + \varphi_4 \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{n}{2}\frac{\varphi_4^2}{\varphi} = 0\ . \tag{25}$$

where suffix '4' indicates differentiation with respect to time.
The energy conservation equation $T^j_{i;j} = 0$ leads to

$$\begin{aligned} &\rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) \\ &\qquad - \left[\frac{\partial}{\partial t}\left(\frac{H^2}{2\overline{\mu} B^2 C^2}\right) + \frac{H^2}{\overline{\mu} B^2 C^2}\left(\frac{B_4}{B} + \frac{C_4}{C}\right)\right] = 0. \end{aligned} \tag{26}$$

3 Solution of Field Equations

The research on exact solutions is based on some physically reasonable restrictions used to simplify the Einstein equations. Equations (21)–(25) are five equations in six unknowns A, B, C, p, ρ, φ. For the complete determination of these unknowns, one more condition is needed. As in the case of general-relativistic cosmologies, the introduction of inhomogeneities into the cosmological equations produces a considerable increase in mathematical difficulty: non-linear partial differential equations must now be solved. To get deterministic model of universe, we have assumed the eigen value σ_1^1 of shear tensor σ_i^j is proportional to expansion scalar θ which leads to

$$A = (BC)^m\,, \tag{27}$$

where m is a constant and we have assumed the proportionality constant as unity.

The motive behind assuming this condition is explained with reference to Thorne [58]; the observations of the velocity-red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 percent [59,60]. To put more precisely, red-shift studies place the limit $\sigma/H \leq 0.3$ on the ratio of shear, σ, to Hubble constant, H, in the neighbourhood of our Galaxy today. Collins et al. [61] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition σ/θ is constant.

Equations (21) and (22) lead to

$$\begin{aligned}\frac{B_{44}}{B} - \frac{A_{44}}{A} + \frac{C_4 B_4}{CB} - \frac{A_4 C_4}{AC} &= \frac{H^2}{\overline{\mu} B^2 C^2},\\ \frac{B_{44}}{B} - \frac{A_{44}}{A} &= \frac{C_4}{C}(\frac{B_4}{B} - \frac{A_4}{A}) + \frac{H^2}{\overline{\mu} B^2 C^2}.\end{aligned} \tag{28}$$

From equations (22) and (23), we have

$$\begin{aligned}\frac{C_{44}}{C} - \frac{B_{44}}{B} + \frac{C_4 A_4}{CA} - \frac{A_4 B_4}{AB} &= 0,\\ \frac{C_{44}}{C} - \frac{B_{44}}{B} &= \frac{A_4}{A}\left(\frac{B_4}{B} - \frac{C_4}{C}\right).\end{aligned} \tag{29}$$

Using the condition $A = (BC)^m$, equation (29) leads to

$$\frac{C_{44}}{C} - \frac{B_{44}}{B} = m\left(\frac{B_4}{B} + \frac{C_4}{C}\right)\left(\frac{B_4}{B} - \frac{C_4}{C}\right),$$

which leads to

$$\frac{(CB_4 - BC_4)_4}{CB_4 - BC_4} = -\frac{m(BC)_4}{BC}.$$

On integration, it leads to

$$C^2\left(\frac{B}{C}\right)_4 = \frac{L}{(BC)^m}, \tag{30}$$

where L is a constant of integration.

Let us assume

$$BC = \mu\gamma \ , \tag{31}$$

$$\frac{B}{C} = \gamma \qquad \text{(Bali and Chandnani [51]).} \tag{32}$$

Using equations (31) and (32), equation (30) leads to

$$\frac{\gamma_4}{\gamma} = L\mu^{-(m+1)} \ , \tag{33}$$

which leads to

$$\frac{\gamma_{44}}{\gamma} = -(m+1)L\mu^{-(m+2)}\mu_4 + L^2\mu^{-2(m+1)} \ . \tag{34}$$

Using (27) leads to

$$\frac{A_4}{A} = m\left(\frac{B_4}{B} + \frac{C_4}{C}\right), \qquad \frac{A_4}{A} = m\frac{\mu_4}{\mu}. \tag{35}$$

Using equations (33) and (34), equation (28) implies

$$(1-2m)\frac{\mu_{44}}{\mu} + m(1-2m)\frac{\mu_4^2}{\mu^2} + \frac{\gamma_{44}}{\gamma} - \frac{\gamma_4^2}{\gamma^2} + (1+m)\frac{\mu_4\gamma_4}{\mu\gamma} = \frac{2H^2}{\overline{\mu}\mu^2}. \tag{36}$$

Using equations (33) and (34) in equation (36), we get

$$(1-2m)\frac{\mu_{44}}{\mu} + m(1-2m)\frac{\mu_4^2}{\mu^2} = \frac{2H^2}{\overline{\mu}\mu^2}, \tag{37}$$

which leads to

$$2(1-2m)\frac{\mu_{44}}{\mu} + 2m(1-2m)\frac{\mu_4^2}{\mu^2} = \frac{4k}{\mu}, \tag{38}$$

where $k = H^2/\overline{\mu}$.

Let us assume that $\mu_4 = f(\mu)$. Therefore, $\mu_{44} = ff'$, where $f' = \frac{df}{d\mu}$.

Equation (38) leads to

$$\frac{df^2}{d\mu} + \frac{2m}{\mu}f^2 = \frac{4k}{(1-2m)\mu},$$

which again leads to

$$f^2 = \frac{2k}{m(1-2m)} + \alpha\frac{1}{\mu^{2m}}, \tag{39}$$

where α is constant of integration.

$$f^2 = \beta + \alpha\frac{1}{\mu^{2m}}, \Rightarrow f = \left[\beta + \alpha\frac{1}{\mu^{2m}}\right]^{1/2}, \tag{40}$$

where $\beta = \dfrac{2k}{m(1-2m)}$.

Using equations (33) and (40), we have

$$\gamma = R\left[\frac{1}{\mu^m} + \sqrt{\frac{\beta}{\alpha} + \frac{1}{\mu^{2m}}}\right]^{-L/(m\sqrt{\alpha})}. \tag{41}$$

Thus, we have $A = \mu^m$, i.e.

$$A^2 = \mu^{2m}, \tag{42}$$

$$B^2 = R\mu\left[\frac{1}{\mu^m} + \sqrt{\frac{\beta}{\alpha} + \frac{1}{\mu^{2m}}}\right]^{-L/(m\sqrt{\alpha})}, \tag{43}$$

$$C^2 = \frac{\mu}{R}\left[\frac{1}{\mu^m} + \sqrt{\frac{\beta}{\alpha} + \frac{1}{\mu^{2m}}}\right]^{-L/(m\sqrt{\alpha})}. \tag{44}$$

Hence the metric becomes

$$ds^2 = -\Big(\frac{dt}{d\mu}\Big)^2 d\mu^2 + \mu^{2m}dx^2 + R\mu\Big[\frac{1}{\mu^m} + \sqrt{\frac{\beta}{\alpha} + \frac{1}{\mu^{2m}}}\Big]^{-L/(m\sqrt{\alpha})}dy^2 + \frac{\mu}{R}\Big[\frac{1}{\mu^m} + \sqrt{\frac{\beta}{\alpha} + \frac{1}{\mu^{2m}}}\Big]^{-L/(m\sqrt{\alpha})}dz^2. \tag{45}$$

Now using the transformation $\mu = T$, $x = X$, $\sqrt{R}y = Y$, $\dfrac{1}{\sqrt{R}}z = Z$, the metric (45) reduces to the form

$$ds^2 = -\frac{dT^2}{(\beta + \alpha T^{-2m})} + T^{2m}dX^2 + T\Big[\frac{1}{T^m} + \sqrt{\frac{\beta}{\alpha} + \frac{1}{T^{2m}}}\Big]^{-L/(m\sqrt{\alpha})}dY^2 + T\Big[\frac{1}{T^m} + \sqrt{\frac{\beta}{\alpha} + \frac{1}{T^{2m}}}\Big]^{-L/(m\sqrt{\alpha})}dZ^2. \tag{46}$$

The model (46) has Cigar type singularity at $T = 0$ when $m > 0$ (MacCallum [62]).

4 Some Physical Parameters

Equation (25) leads to

$$\varphi = \left\{ \log \left[\frac{1}{\mu^m} + \sqrt{\frac{\beta}{\alpha} + \frac{1}{\mu^{2m}}} \right]^{\frac{-(n+2)N}{2m\sqrt{\alpha}}} \right\}^{\frac{2}{n+2}} . \tag{47}$$

From equation (26) and taking the stiff fluid condition, i.e. $p = \rho$,

$$\rho_4 + 2(m+1)\rho \left(\frac{\mu_4}{\mu}\right) - \left[\frac{\partial}{\partial t}\left(\frac{k}{2\mu^2}\right) + \frac{k}{\mu^2}\left(\frac{\mu_4}{\mu}\right)\right] = 0,$$

which leads to

$$\rho = \frac{Q}{[\mu^2]^{m+1}} = p, \tag{48}$$

where Q is the constant of integration .

From equation (48), it is noted that the positive energy density is a positive decreasing function of time and it approaches a small positive value at present epoch. Here we also see that energy density is a decreasing function of time, which shows that the universe is expanding.

The expansion scalar θ is given by

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C},$$

which leads to

$$\theta = \frac{(m+1)}{T^{m+1}} \left(\beta T^{2m} + \alpha\right)^{\frac{1}{2}} . \tag{49}$$

Components of the shear tensor σ_i^j are given by

$$\sigma_1^1 = \frac{1}{3}\left(\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right), \tag{50}$$

$$\sigma_2^2 = \frac{1}{3}\left(\frac{2B_4}{B} - \frac{A_4}{A} - \frac{C_4}{C}\right), \tag{51}$$

$$\sigma_3^3 = \frac{1}{3}\left(\frac{2C_4}{C} - \frac{B_4}{B} - \frac{A_4}{A}\right), \tag{52}$$

$$\sigma_4^4 = 0. \tag{53}$$

So we obtain

$$\sigma_1^1 = \left(\frac{2m-1}{3}\right)\left[\frac{(\beta T^{2m}+\alpha)^{1/2}}{T^{m+1}}\right], \tag{54}$$

$$\sigma_2^2 = \frac{1}{6T^{m+1}}\left\{(1-2m)\left(\beta T^{2m}+\alpha\right)^{1/2}+3L\right\}, \tag{55}$$

$$\sigma_3^3 = \frac{1}{6T^{m+1}}\left\{(1-2m)\left(\beta T^{2m}+\alpha\right)^{1/2}-3L\right\}, \tag{56}$$

$$\sigma_4^4 = 0. \tag{57}$$

The shear σ is given by

$$\sigma^2 = \frac{1}{2}\left[\left(\sigma_1^1\right)^2+\left(\sigma_2^2\right)^2+\left(\sigma_3^3\right)^2+\left(\sigma_4^4\right)^2\right],$$

which leads to

$$\begin{aligned}\sigma^2 &= \frac{1}{4T^{2(m+1)}}\left\{\frac{(2m-1)^2\left(\beta T^{2m}+\alpha\right)+3L^2}{3}\right\},\\ \sigma &= \frac{1}{2\sqrt{3}T^{(m+1)}}\left[(2m-1)^2\left(\beta T^{2m}+\alpha\right)+3L^2\right]^{1/2}.\end{aligned} \tag{58}$$

The spatial volume $\left(R^3\right)$

$$\begin{aligned}R^3 &= ABC\\ &= (BC)^{m+1}\\ R^3 &= T^{m+1}.\end{aligned} \tag{59}$$

From the above results, it can be seen that the spatial volume is zero at $T=0$ and it increases with the increase of T. This shows that the universe starts evolving with zero volume at $T=0$ and expands with cosmic time T.

Thus, we find

$$\frac{\sigma_1^1}{\theta} = \frac{2m-1}{3(m+1)}, \tag{60}$$

which is constant. Hence the anisotropy is maintained throughout.

The deceleration parameter q is given by

$$q = \frac{\frac{-R_{44}}{R}}{\frac{R_4^2}{R^2}}. \tag{61}$$

This leads to

$$q = \frac{-\frac{(m+1)^2}{9}\frac{[\beta T^{2m}+\alpha]}{T^{2(m+1)}}}{\frac{m+1}{3}\left[\frac{-\alpha m}{T^{m+1}} + m\beta T^{m-1} + \frac{\alpha m}{T^{m+1}}\right] - \frac{2(m+1)^2}{9}\left[\frac{\beta}{T} + \frac{\alpha}{T^{2(m+1)}}\right]}. \quad (62)$$

The negative value of deceleration parameter 'q' implies that our model (46) of universe is accelerating. The value of q is consistent with the current observations [63].

5 Solution in the Absence of Magnetic Field

To find the solution in the absence of magnetic field, we put $k = 0$ in equation (38) and get

$$2(1-2m)\frac{\mu_{44}}{\mu} + 2m(1-2m)\frac{\mu_4^2}{\mu^2} = 0, \implies \frac{\mu_{44}}{\mu} + m\frac{\mu_4^2}{\mu^2} = 0, \quad (63)$$

which leads to

$$\mu = (\alpha_1 t + \beta_1)^{\frac{l}{\alpha_1}}, \quad (64)$$

where $\beta_1 = (m+1)b_1$ and $\alpha_1 = (m+1)l$.

Using equations (31) and (32), we get

$$\frac{\gamma_4}{\gamma} = \frac{L}{\mu^{m+1}},$$

which leads to

$$\frac{\gamma_4}{\gamma} = \frac{L}{\alpha_1 t + \beta_1}.$$

Integrating, we have

$$\gamma = D\,(\alpha_1 t + \beta_1)^{\frac{L}{\alpha_1}}, \quad (65)$$

where D is the constant of integration.

Now, $B^2 = \mu\gamma$, which leads to

$$B^2 = D(\alpha_1 t + \beta_1)^{\frac{l+L}{\alpha_1}}, \implies B = \sqrt{D}(\alpha_1 t + \beta_1)^E, \quad (66)$$

where $E = \frac{L+l}{2\alpha_1}$.

Again, $C^2 = \frac{\mu}{\gamma}$, which leads to

$$C^2 = \frac{(\alpha_1 t + \beta_1)^{\frac{l-L}{\alpha_1}}}{D}, \implies C = \frac{(\alpha_1 t + \beta_1)^F}{\sqrt{D}}. \quad (67)$$

Thus, we have $A = \mu^m$,

$$A = (\alpha_1 t + \beta_1)^G \,, \tag{68}$$

where $G = \dfrac{\alpha_1 - l}{\alpha_1}$.

Here the constants E, F, G satisfy the relation

$$E + F + G = 1\,. \tag{69}$$

Hence the metric becomes

$$ds^2 = -dt^2 + (\alpha_1 t + \beta_1)^{2G} dx^2 + D(\alpha_1 t + \beta_1)^{2E} dy^2 + \frac{(\alpha_1 t + \beta_1)^{2F}}{D} dz^2. \tag{70}$$

Now using the transformation $\mu = T$, $x = X$, $\sqrt{D}y = Y$, $\dfrac{1}{\sqrt{D}}z = Z$, the metric (70) reduces to the form

$$ds^2 = -\frac{dT^2}{\alpha_1^2} + T^{2G} dX^2 + T^{2E} dY^2 + T^{2F} dz^2, \tag{71}$$

where $E + F + G = 1$.

The function φ is given by,

$$\varphi = \left\{ \left(\frac{n+2}{2} \right) \frac{I}{\alpha_1} \log\left(\alpha_1 t + \beta_1\right) \right\}^{2/(n+2)} . \tag{72}$$

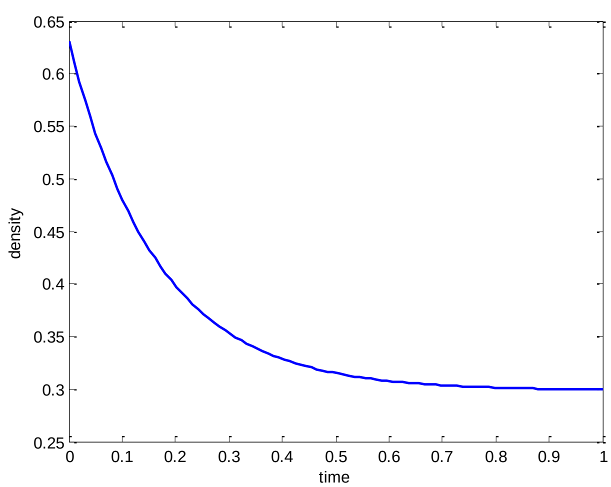

Figure 1.

In this case, the matter density and the isotropic pressure are given by

$$\rho = p = \frac{M}{T^2}. \tag{73}$$

We observe that $\rho(t)$ is a decreasing function of time. The reality conditions given by Ellis [64] as (i) $\rho + p > 0$, (ii) $\rho + 3p > 0$, are satisfied when $M > 0$.
The expansion scalar θ is given by

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C},$$

which leads to

$$\theta = \frac{\alpha_1}{T} \ . \tag{74}$$

For the spatial volume $\left(R^3, wehave\right)$

$$\begin{aligned} R^3 &= ABC \\ &= T^{E+F+G} \\ &= T. \end{aligned} \tag{75}$$

Components of the shear tensor σ_i^j are given by

$$\begin{aligned} \sigma_1^1 &= \frac{1}{3}\left(\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right), \\ \sigma_2^2 &= \frac{1}{3}\left(\frac{2B_4}{B} - \frac{A_4}{A} - \frac{C_4}{C}\right), \\ \sigma_3^3 &= \frac{1}{3}\left(\frac{2C_4}{C} - \frac{B_4}{B} - \frac{A_4}{A}\right), \\ \sigma_4^4 &= 0. \end{aligned}$$

So we obtain

$$\sigma_1^1 = \left(G - \frac{1}{3}\right)\left[\frac{\alpha_1}{T}\right], \tag{76}$$

$$\sigma_2^2 = \left(E - \frac{1}{3}\right)\left[\frac{\alpha_1}{T}\right], \tag{77}$$

$$\sigma_3^3 = \left(F - \frac{1}{3}\right)\left[\frac{\alpha_1}{T}\right], \tag{78}$$

$$\sigma_4^4 = 0. \tag{79}$$

The shear σ is given by

$$\sigma^2 = \frac{1}{2}\left[\left(\sigma_1^1\right)^2 + \left(\sigma_2^2\right)^2 + \left(\sigma_3^3\right)^2 + \left(\sigma_4^4\right)^2\right],$$

which leads to

$$\sigma^2 = \frac{1}{2}\left\{\left(G - \frac{1}{3}\right)^2 + \left(E - \frac{1}{3}\right)^2 + \left(F - \frac{1}{3}\right)^2\right\}\frac{\alpha_1^2}{T^2}. \tag{80}$$

Thus, we find

$$\frac{\sigma_1^1}{\theta} = G - \frac{1}{3} \neq 0, \tag{81}$$

which is constant. Hence in the absence of magnetic field the anisotropy is maintained throughout.

The deceleration parameter q is given by

$$q = \frac{\frac{-R_{44}}{R}}{\frac{R_4^2}{R^2}},$$

therefore,

$$q = 2. \tag{82}$$

Observations by the differential Radiometers on NACA's Cosmic Background Explorer registered anisotropy in various angles scales. It is conjectured, that these anisotropies hide in their hearts the entire history of the cosmic evolution down to recombination, and they are considered to be indicative of the universe geometry and the matter composing the universe. The theoretical arguments [65] and the modern experimental data support the existence of an anisotropic phase, which turns into an isotropic one [66]. The deceleration parameter is positive, i.e. the universe was decelerating at the time of inflation. This is in accordance with modern cosmological observations [67-69]. To indicate whether a model inflates or not one can see the sign of q. A negative sign $-1 \leq q < 0$, indicates inflations whereas a positive sign corresponds to the standard decelerating model. Recent observations [70,71] show that the deceleration parameter of the Universe is in the range $-1 \leq q < 0$ and the present day Universe is undergoing an accelerated expansion. From (82), it is interesting to note that in the absence of electromagnetic field, the expansion of the Universe is always decelerating which is a contradictory result and hence we conclude that electromagnetic field plays an important role on the evolution of the Universe. It is interesting to note that our results resemble with the results obtained by H. Amirashchi [72].

Now,

$$\int_{t_0}^{t} \frac{dt}{R(t)} = \frac{3}{2}(t - t_0) \tag{83}$$

is finite.

The integral is convergent, so particle horizon exists.

6 Conclusion

The model (46) starts with a big-bang at $T = 0$ in the presence of magnetic field. The model (46) has Cigar type singularity at $T = 0$ (MacCallum [62]). The scalar field is large initially but it decreases as time increases. The energy density $\rho \to 0$ and pressure $p \to 0$ when $T \to \infty$ and $\rho \to \infty$, $p \to \infty$ when $T \to 0$. The spatial volume increases as time increases. Thus, the inflationary scenario exists. Since $\sigma/\theta \neq 0$, hence the model (46) represents an anisotropic universe.

In the absence of magnetic field, the model (71) starts with a big-bang at $T = 0$ and the expansion in the model decreases as time increases. It ceases out when $T \to \infty$. Since deceleration parameter $q > 0$, we find that the model (71) represents a decelerating universe. The spatial volume increases as time increases. The scalar field decreases as time increases. It is interesting to note that in the presence of magnetic field, our results resemble to Bali and Chandnani [51] and in the absence of magnetic field our results resemble to Bali and Chandnani [73].

References

[1] C. Brans and R.H. Dicke (1961) *Phys. Rev*. **124** 925.

[2] K. Nordvedt (1970) *Astrophys. J*. **161** 1059.

[3] G.A. Barber (1982) *Gen. Relativ. Gravit*. **14** 117.

[4] D. Saez and V.J. Ballester (1985) *Phys. Lett.* **A113** 467.

[5] Y.K. Lau and S.J. Prokhovnik (1986) *Aust. J. Phys*. **39** 339.

[6] T. Singh, A.K. Agrawal (1991) *Astrophys. Space Sci*. **182** 289.

[7] T. Singh, A.K. Agrawal (1992) *Astrophys. Space Sci*. **191** 61.

[8] S. Ram, S.K. Tiwari (1998) *Astrophys. Space Sci*. **259** 91.

[9] C.P. Singh, S. Ram (2003) *Astrophys. Space Sci*. **284** 1999.

[10] G. Mohanty, S.K. Sahu (2003) *Astrophys. Space Sci*. **288** 509.

[11] G. Mohanty, S.K. Sahu (2004) *Astrophys. Space Sci*. **291** 75.

[12] S.K. Tripathi, S.S.K. Nayak, S.K. Sahu, T.R. Routray (2009) *Int. J. Theor. Phys*. **48** 213.

[13] D.R.K. Reddy, R.L. Naidu, V.U.M. Rao (2006) *Astrophys. Space Sci*. **306** 185.

[14] D.R.K. Reddy, P. Govinda, R.L. Naidu (2008) *Int. J. Theor. Phys*. **47** 2966.

[15] D.R.K. Reddy, R.L. Naidu (2007) *Astrophys. Space Sci*. **312** 99.

[16] V.U.M. Rao, T. Vinutha, M.V. Shanthi, G.S.D. Kumari (2008) *Astrophys. Space Sci*. **317** 89.

[17] V.U.M. Rao, M.V. Shanthi, T. Vinutha (2008) *Astrophys. Space Sci*. **317** 27.

[18] V.U.M. Rao, G.S.D. Kumari, K.V.S. Sireesha (2011) *Astrophys. Space Sci*. **335** 635.

[19] K.S. Adhav, M.R. Ugale, C.B. Kale, M.P. Bhende (2007) *Int. J. Theor. Phys*. **46** 3122.

[20] S.D. Katore, K.S. Adhav, A.Y. Shaikh, N.K. Sarkate (2010) *Int. J. Theor. Phys*. **49** 2558.

[21] S.K. Sahu (2010) *J. Mod. Phys*. **1** 67.
[22] C.P. Singh (2009) *Braz. J. Phys*. **39** 669.
[23] A. Pradhan, S.K. Singh (2010) *Electron. J. Theor. Phys*. **7** 407.
[24] J. Socorro, M. Sabido (2010) *Rev. Mex. Fis*. **56** 166.
[25] M. Jamil, S. Ali, D. Momeni, R. Myrzakulov (2012) *Eur. Phys. J. C* **72** 1998; arXiv:1201.0895.[physics.gen-ph]
[26] R.L. Naidu, B. Satyanarayana, D.R.K. Reddy (2012) *Astrophys. Space Sci*. **338** 333.
[27] R.L. Naidu, B. Satyanarayana, D.R.K. Reddy (2012) *Astrophys. Space Sci*. **338** 351.
[28] D.R.K. Reddy, B. Satyanarayana, R.L. Naidu (2012) *Astrophys. Space Sci*. **339** 401.
[29] Ya.B. Zel'dovich (1993) "Magnetic Field in Astrophysics", Gordon and Breach, New York.
[30] E.R. Harrison (1973) *Phys. Rev. Lett*. **30** 188.
[31] C.W. Misner, K.S. Thorne, J.A. Wheeler (1973) "Gravitation", Freeman, New York.
[32] R. Perley, G. Taylor (1991) *Astrophys. J*. **101** 1623.
[33] R. Pudritz, J. Silk (1989) *Astrophys. J*. **342** 650.
[34] A. Banerjee, A.K. Sanyal, S. Chakraborty (1990) *Pramana J. Phys*. **34** 1.
[35] S. Chakraborty (1980) *Int. J. Pure Appl. Phys*. **29** 31.
[36] R. Tikekar, L.K. Patel (1992) *Gen. Relativ. Gravit*. **24** 397.
[37] R. Tikekar, L.K. Patel (1994) *Pramana J. Phys*. **42** 483.
[38] R. Bali, R.D. Upadhaya (2003) *Astrophys. Space Sci*. **283** 97.
[39] R. Bali, S. Dave (2001) *Pramana J. Phys*. **56** 513.
[40] R. Bali, D.K. Singh (2005) *Astrophys. Space Sci*. **300** 387.
[41] R. Bali, Anjali (2006) *Astrophys. Space Sci*. **302** 201.
[42] R. Bali, A. Pradhan, H. Amirhashchi (2008) *Int. J. Theor. Phys*. **47** 2549.
[43] R. Bali, A. Pradhan (2007) *Chin. Phys. Lett*. **24** 85.
[44] M.K. Yadav, A. Pradhan, S.K. Singh (2007) *Astrophys. Space Sci*. **311** 423.
[45] M.K. Yadav, A. Rai, A. Pradhan (2007) *Int. J. Theor. Phys*. **46** 2677.
[46] A. Pradhan, P. Mathur (2008) *Astrophys. Space Sci*. **318** 255.
[47] A. Pradhan, A. Rai, S.K. Singh (2007) *Astrophys. Space Sci*. **312** 261.
[48] A. Pradhan, S.K. Srivastav, K. Jotania (2004) *Czech. J. Phys*. **54** 255.
[49] A. Pradhan, P. Pandey, K.K. Rai (2006) *Czech. J. Phys*. **56** 303.
[50] H.P. Robertson and A.G. Walker (1936) *Proc. London Math. Soc*. **42** 90.
[51] R. Bali, N.R. Chandani (2009) *Fizika B* **18** 227-242.
[52] R.S. Singh and A. Singh (20120 *EJTP* **9** 251-268.
[53] R. Bali and R. Vadhwani (2011) *Int. J. Phys. Sci.* **6** 6171-6178.
[54] R. Bali, N.K. Chandnani and L.K. Gupta (2010) *Commun. Theor. Phys.* **54** 197.
[55] A. Lichnerowicz (1967) " Relativistic Hydrodynamics and Magnetohydrodynamics", Benjamin, New York, p. 13.
[56] R. Maartens (2000) *Pramana J. Phys.* **55** 557-583.
[57] K.A. Bronnikov et al. (2004) *Class. Quantum Grav*. **21** 3389-3403.
[58] K.S. Thorne (1967) *Astrophys. J.* **148** 51.
[59] R. Kantowski and R.K. Sachs (1966) *J. Math. Phys.* **7** 443-446.
[60] J. Kristian and R.K. Sachs (1966) *Astrophys. J.* **143** 379-399.

[61] C.B. Collins et al. (1980) *Gen. Relativ. Gravit*. **12** 805.
[62] M.A.H. MacCallum (1971) *Commun. Math. Phys.* **20** 57.
[63] R.A. Knop, et al. (2003) *Astrophys. J*. **598** 102.
[64] G.F.R. Ellis (1971) “General Relativity and Cosmology , ed. R.K. Sachs, Clarendon Press, p. 117.
[65] C.W. Misner (1968) *Astrophys. J.* **151** 431-457.
[66] B. Saha (2009) *Phys. Part. Nuclei* **40** 656-673.
[67] S. Perlmutter et al. (1997) *Astrophys. J.* **517** 565.
[68] A.G. Riess et al. (1998) *Astrophys. J.* **116** 1009.
[69] P.M. Garnavich et al. (1998) *Astrophys. J.* **493** L53.
[70] A.G. Riess et al. (2004) *Astrophys. J*. **607** 665.
[71] S. Perlmutter et al., *Astrophys. J*. **517** 565.
[72] H. Amirhashchi (2013) *Pramana J. Phys.* **80** 723-738.
[73] R. Bali and N.R. Chandnani (2008) *J. Math. Phys.* **49** 032502.