Shock Waves in a Dusty Plasma with Positive and Negative Dust, where Electrons Are Superthermally Distributed

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Abstract. Using the reductive perturbation technique, Burgers’ equation is derived for studying the behavior of dust acoustic (DA) shock waves in an unmagnetized dusty plasma, whose constituents are kappa-distributed (superthermal) electrons, Boltzmann-distributed ions and positively and negatively charged dust particles. The solution of Burgers’ equation is numerically analyzed. The effects of spectral index \( \kappa \) on the properties of DA shock waves is discussed. It is found that the spectral index \( \kappa \) changes the amplitude of the DA shock waves significantly. The present investigation may be helpful in the understanding of the nonlinear propagation of waves in the pulsar magnetosphere.

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1 Introduction

Dusty plasma research is one of the most rapidly growing branches of plasma physics. The presence of dusty plasmas in comet tails, asteroid zones, planetary ring, interstellar medium, lower part of the earth’s ionosphere and the magnetosphere makes this subject rapidly important. Dusty plasma plays a vital role to understanding different types of new and interesting aspects in field like low temperature physics, radiofrequency plasma discharge, plasma crystals etc. Such plasmas are investigated theoretically and experimentally also [1-6]

Dusty plasma supports mainly two types of acoustic waves: high frequency dust ion acoustic wave (DIAWs) involving mobile ions and static dust grains, and a low frequency dust acoustic wave (DAWs) involving mobile dust grains. Both of these modes have been studied theoretically [7-9] and experimentally [10,11].
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Usually, the Reductive Perturbation Technique (RPT) is applied to study solitary waves. The discoveries of dust acoustic wave (DAW) give a new impact to study the waves in dusty plasma [12,13]. In most of the studies on dusty plasmas, the dust grains are assumed to be negatively charged. However, this assumption is questionable in different realistic situations. V.E. Fortov [14] has shown there are three mechanisms by which the dust grains can be positively charged. These three mechanisms are photoemission induced by radiative heating and secondary emission of electrons from the surface of the dust grains. Recently positively charged dust grains are found in space plasma environments and it is seen that these positively charged grains also play significant role. G. Mandal et al. [15] studied dusty plasmas taking into account both positive and negative dust grains. They found negative as well as positive shock potentials (for electron and ions).

Armina et al. [16] have considered a four-component dusty plasma containing positively and negatively charged dust grain and Boltzmann distributed electrons and ions. They investigated the possibility for the formation of shock waves and the existence of shock structures.

Most of the research papers which discussed DASWs are based on electrons and ions with Maxwell-Boltzmann distributions. In collisionless plasma, however, particle velocity distributions can often depart from being Maxwellian. For example, in naturally occurring plasma in the planetary magnetosphere and in solar wind, the particle velocity distribution is observed to have non-Maxwellian (power-law), high-energy tail [17]. Such distribution may be accurately modelled by a Kappa (or Generalized Lorentzian) distribution [18]. A three-dimensional generalized Lorentzian or \( \kappa \)-distribution function takes the form

\[
F_{\kappa}(v) = A_{\kappa} \left[ 1 + \frac{\theta^2}{\kappa B^2} \right]^{-(\kappa+1)},
\]

where \( A_{\kappa} \) is the normalization constant, \( \theta = \left[ \frac{(\kappa - 3/2) / \kappa} {v_{th}^2} \right]^{1/2} \); \( v_{th} = (2k_B T/m)^{1/2} \) is the effective thermal speed related to the usual thermal speed, and \( \kappa \) is the spectral index. The spectral index \( \kappa \) is a measure of the slope of the energy spectrum of the suprathermal particles forming the tail of the velocity distribution function. Kappa \((\kappa)\) distribution approaches to Maxwellian as \( \kappa \) tends to infinity. However, a few observations in space plasmas [19,20] indicate clearly the presence of superthermal (kappa-distributed) electron and ion structures in a variety of astrophysical plasma environments.

Recently A. Shah et al. [21] have studied the shock wave structure with kappa-distributed electrons and positrons. They derived Korteweg-de Vries (KdV)-Burgers’ equation and solved it analytically. The effect of plasma parameters on the shock strength and steepness are investigated. Recently B. Sahu [22] has investigated the cylindrical or spherical dust-ion acoustic shock waves in an adiabatically dusty plasma and found that the nonplanar geometry effects have a very significant role in the formation of shock waves. In this paper, we study a four
component unmagnetized dusty plasma comprising of Boltzmann-distributed ions, Kappa-distributed electrons and positively and negatively charged dust grains. We have been particularly interested to observe the effect of the superthermal electrons, i.e., the effect of spectral index $\kappa$ on the DA shock waves in our four component dusty plasma system.

The paper is designed as follows: basic equations associated with the plasma model are given in Section 2 and the Burgers’ equation is derived using the RPT in Section 3. Solutions, results, and discussions are highlighted in Section 4. Section 5 is kept for conclusions.

## 2 Governing Equations

We consider one-dimensional, unmagnetized, collisionless dusty plasma consisting of superthermal electrons, Boltzmann-distributed ions and positively and negatively charged dust grains. The nonlinear dynamics of DA waves is governed by

\[
\frac{\partial n_1}{\partial t} + \frac{\partial (n_1 u_1)}{\partial x} = 0, \quad (1)
\]

\[
\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} = \frac{z_1 e}{n_1} \frac{\partial \phi}{\partial x} + \eta_1 \frac{\partial^2 u_1}{\partial x^2}, \quad (2)
\]

\[
\frac{\partial n_2}{\partial t} + \frac{\partial (n_2 u_2)}{\partial x} = 0, \quad (3)
\]

\[
\frac{\partial u_2}{\partial t} + u_2 \frac{\partial u_2}{\partial x} = -\frac{z_2 e}{n_2} \frac{\partial \phi}{\partial x} + \eta_1 \frac{\partial^2 u_2}{\partial x^2}, \quad (4)
\]

\[
n_e - n_i + z_1 n_1 - z_2 n_2 = 0, \quad (5)
\]

where $n_1$ ($n_2$) is the negative (positive) dust number density, $u_1$ ($u_2$) is the negative (positive) dust fluid speed, $z_1$ ($z_2$) is the number of electrons (protons) residing on a negative (positive) dust particle, $-e$ is the electron charge, $\phi$ is the wave potential, $\eta_1$ ($\eta_1$) is the viscosity coefficient of negative (positive) dust fluid, $x$ is the space variable, and $t$ is the time variable, $n_e$ ($n_i$) is the electron (ion) number density:

\[
n_e = n_{e0} \left(1 - \frac{e\phi}{k_B T_e} \right)^{-\kappa - \frac{1}{2}}, \quad n_i = n_{i0} \exp \left(-\frac{e\phi}{k_B T_i} \right).
\]

Here $T_e$ ($T_i$) is the temperature of electrons (ions), and $k_B$ is the Boltzmann constant. In Eq. (5) we have assumed the quasi-neutrality condition at equilibrium.

Now, in terms of normalized variables, namely

\[
N_1 = \frac{n_1}{n_{10}}, \quad N_2 = \frac{n_2}{n_{20}}, \quad U_1 = \frac{u_1}{c_1}, \quad U_2 = \frac{u_2}{c_1},
\]

\[
\psi = \frac{e\phi}{k_B T_i}, \quad T = tw_{pd}, \quad X = \frac{x}{\lambda_D}, \quad \eta_1 = \eta_{1w_{pd}} \lambda_D^2, \quad \eta_2 = \eta_{1w_{pd}} \lambda_D^2.
\]

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where \( n_{10} (n_{20}) \) is the equilibrium value of \( n_1 (n_2) \),

\[
c_1 = \sqrt{\frac{z_1 k_B T_i}{m_1}}, \quad w_{pd} = \sqrt{\frac{4 \pi z_1^2 e^2 n_{10}}{m_1}}, \quad \lambda_{D} = \sqrt{\frac{z_1 k_B T_i}{4 \pi z_1^2 e^2 n_{10}}},
\]

we can express Eqs. (1)–(5) as

\[
\frac{\partial N_1}{\partial T} + \frac{\partial (N_1 U_1)}{\partial X} = 0, \quad (6)
\]

\[
\frac{\partial U_1}{\partial T} + \frac{U_1}{\partial X} \frac{dU_1}{dX} = \frac{\partial \psi}{\partial X} + \eta_1 \frac{\partial^2 U_1}{\partial X^2}, \quad (7)
\]

\[
\frac{\partial N_2}{\partial T} + \frac{\partial N_2 U_2}{\partial X} = 0, \quad (8)
\]

\[
\frac{\partial U_2}{\partial T} + \frac{U_2}{\partial X} \frac{dU_2}{dX} = -\alpha \beta \frac{\partial \psi}{\partial X} + \eta_2 \frac{\partial^2 U_2}{\partial X^2}, \quad (9)
\]

\[
N_1 = (1 + \mu_e - \mu_i) N_2 - \mu_e \left( 1 - \frac{\sigma \psi}{\kappa - \frac{\psi}{2}} \right)^{-\kappa - \frac{\psi}{2}} + \mu_i \exp(-\psi), \quad (10)
\]

where

\[
\alpha = \frac{z_2}{z_1}, \quad \beta = \frac{m_1}{m_2}, \quad \mu_e = \frac{n_{e0}}{z_1 n_{10}}, \quad \mu_i = \frac{n_{i0}}{z_1 n_{10}}, \quad \sigma = \frac{T_i}{T_e}.
\]

### 3 Derivation of Burgers’ Equation

Now, we derive the Burgers’ equation from Eqs. (6)–(10) by employing the reductive perturbation technique (RPT) and the stretched coordinates \( \xi = \epsilon^{1/2} (X - V_0 T) \) and \( \tau = \epsilon^{3/2} T \) [23], where \( \epsilon \) is a smallness parameter measuring the weakness of the nonlinearity and \( V_0 \) is the phase speed of the DA waves normalized by \( c_1 \), we now express (6) to (10) in terms of \( \xi \) and \( \tau \) as

\[
\epsilon^{3/2} \frac{\partial N_1}{\partial \tau} - V_0 \epsilon^{1/2} \frac{\partial N_1}{\partial \xi} + \epsilon^{3/2} \frac{\partial (N_1 U_1)}{\partial \xi} = 0, \quad (11)
\]

\[
\epsilon^{3/2} \frac{\partial U_1}{\partial \tau} - V_0 \epsilon^{1/2} \frac{\partial U_1}{\partial \xi} + \epsilon^{3/2} \frac{\partial U_1}{\partial \xi} = \epsilon^{3/2} \frac{\partial \psi}{\partial \xi} + \epsilon^{3/2} \eta_{10} \frac{\partial^2 U_1}{\partial \xi^2}, \quad (12)
\]

\[
\epsilon^{3/2} \frac{\partial N_2}{\partial \tau} - V_0 \epsilon^{1/2} \frac{\partial N_2}{\partial \xi} + \epsilon^{3/2} \frac{\partial (N_2 U_2)}{\partial \xi} = 0, \quad (13)
\]

\[
\epsilon^{3/2} \frac{\partial U_2}{\partial \tau} - V_0 \epsilon^{1/2} \frac{\partial U_2}{\partial \xi} + \epsilon^{3/2} \frac{\partial U_2}{\partial \xi} = -\epsilon^{3/2} \alpha \beta \frac{\partial \psi}{\partial \xi} + \epsilon^{3/2} \eta_{20} \frac{\partial^2 U_2}{\partial \xi^2}, \quad (14)
\]

\[
N_1 = (1 + \mu_e - \mu_i) N_2 - \mu_e \left( 1 - \frac{\sigma \psi}{\kappa - \frac{\psi}{2}} \right)^{-\kappa - \frac{\psi}{2}} + \mu_i e^{-\psi}, \quad (15)
\]

where \( \eta_1 = \epsilon^{3/2} \eta_{10} \) and \( \eta_2 = \epsilon^{3/2} \eta_{20} \) are assumed.
We can expand the variables $N_1, U_1, N_2, U_2$ and $\psi$ in a power series of $\epsilon$ as

\begin{align*}
N_1 &= 1 + \epsilon N_1^{(1)} + \epsilon^2 N_1^{(2)} + \cdots \\
U_1 &= 0 + \epsilon U_1^{(1)} + \epsilon^2 U_1^{(2)} + \cdots \\
N_2 &= 1 + \epsilon N_2^{(1)} + \epsilon^2 N_2^{(2)} + \cdots \\
U_2 &= 0 + \epsilon U_2^{(1)} + \epsilon^2 U_2^{(2)} + \cdots \\
\psi &= 0 + \epsilon \psi^{(1)} + \epsilon^2 \psi^{(2)} + \cdots
\end{align*}

(16)–(20)

Now substituting (16)–(20) into (11)–(15) and taking the coefficient of $\epsilon^{3/2}$ from (11)–(14) and $\epsilon$ from (15), we get

\begin{align*}
N_1^{(1)} &= \frac{U_1^{(1)}}{V_0} \\
U_1^{(1)} &= -\frac{\psi^{(1)}}{V_0} \\
N_2^{(1)} &= \frac{U_2^{(1)}}{V_0} \\
U_2^{(1)} &= (\alpha \beta) \frac{\psi^{(1)}}{V_0} \\
N_1^{(1)} &= (1 + \mu_e - \mu_i) N_2^{(1)} - \sigma \mu_e \frac{\kappa + \frac{1}{2}}{\kappa - \frac{1}{2}} \psi^{(1)} - \mu_i \psi^{(1)}.
\end{align*}

(21)–(25)

Now using (21)–(25), we get

\begin{align*}
N_1^{(1)} &= -\frac{\psi^{(1)}}{V_0} \\
N_2^{(1)} &= (\alpha \beta) \frac{\psi^{(1)}}{V_0^2} \\
V_0^2 &= \frac{(\kappa - \frac{1}{2}) [1 + \alpha \beta (1 + \mu_e - \mu_i)]}{[\sigma \mu_e (\kappa + \frac{1}{2}) + \mu_i (\kappa - \frac{1}{2})]}. \\
\end{align*}

(26)–(28)

Equation (28) is the linear dispersion relation for the DA shock waves propagating in our dusty plasma system. Similarly, substituting (16)–(20) into (11)–(15) and equating the coefficient of $\epsilon^{3/2}$ from (11)–(14) and $\epsilon^2$ from (15), one obtains

\begin{align*}
\frac{\partial N_1^{(1)}}{\partial \tau} - V_0 \frac{\partial N_1^{(2)}}{\partial \xi} + \frac{\partial U_1^{(2)}}{\partial \xi} + \frac{\partial (N_1^{(1)} U_1^{(1)})}{\partial \xi} &= 0 \\
\frac{\partial U_1^{(1)}}{\partial \tau} - V_0 \frac{\partial U_1^{(2)}}{\partial \xi} + U_1^{(1)} \frac{\partial U_1^{(1)}}{\partial \xi} &= \frac{\partial \psi^{(2)}}{\partial \xi} + \eta \frac{\partial^2 U_1^{(1)}}{\partial \xi^2}
\end{align*}

(29)–(30)
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\[
\frac{\partial N_2^{(2)}}{\partial \tau} - V_0 \frac{\partial u_2^{(2)}}{\partial \xi} + \frac{\partial \left(N_2^{(1)} u_2^{(1)}\right)}{\partial \xi} = 0
\]  

(31)

\[
\frac{\partial u_2^{(1)}}{\partial \tau} - V_0 \frac{\partial u_2^{(2)}}{\partial \xi} + u_2^{(1)} \frac{\partial u_2^{(1)}}{\partial \xi} = -\alpha \beta \frac{\partial \psi^{(2)}}{\partial \xi} + \eta_20 \frac{\partial^2 u_2^{(1)}}{\partial \xi^2}
\]  

(32)

\[N_1^{(2)} = (1 + \mu_e - \mu_i) N_2^{(2)} - \mu_e \left[ \frac{\kappa + \frac{1}{2}}{\kappa - \frac{1}{2}} \psi^{(2)} \right.
\]

\[+ \sigma^2 \frac{(\kappa + \frac{1}{2})(\kappa + \frac{3}{2})}{2(\kappa - \frac{1}{2})^2} \left( \psi^{(1)} \right)^2 \]  

\[+ \mu_i \left[ - \psi^{(2)} + \frac{1}{2} \left( \psi^{(1)} \right)^2 \right] \]  

(33)

Now, using (22), (24) and (26)–(28), and eliminating \(N_1^{(2)}\), \(N_2^{(2)}\), \(U_1^{(2)}\), \(U_2^{(2)}\), and \(\psi^{(2)}\) from the above set of equations, we finally obtain

\[
\frac{\partial \psi^{(1)}}{\partial \tau} + A \psi^{(1)} \frac{\partial \psi^{(1)}}{\partial \xi} = C \frac{\partial^2 \psi^{(1)}}{\partial \xi^2},
\]  

(34)

where the nonlinear coefficient \(A\) and the dissipation coefficient \(C\) are given by

\[
A = \left\{ \left( \kappa - \frac{1}{2} \right)^2 \left[ 3\alpha^2 \beta^2 (1 + \mu_2 - \mu_i) - 3 \right] \right.
\]

\[- V_0^2 \left[ \sigma^2 \mu_e \left( \kappa + \frac{1}{2} \right) \left( \kappa + \frac{3}{2} \right) - \mu_i \left( \kappa - \frac{1}{2} \right)^2 \right] \}
\]

\[\times \left\{ 2V_0 \left( \kappa - \frac{1}{2} \right)^2 \left[ 1 + \alpha \beta (1 + \mu_e - \mu_i) \right] \right\}^{-1}
\]  

(35)

\[
C = \frac{\eta_1 + \eta_20 \alpha \beta (1 + \mu_2 - \mu_i)}{2[1 + \alpha \beta (1 + \mu_e - \mu_i)]}.
\]  

(36)

Equation (34) is known as Burgers’ equation which can describe the nonlinear propagation of DA shock waves in our four component dusty plasma system.

4 Numerical Results and Discussions

The stationary solution of this Burgers’ equation is obtained by transforming the independent variables \(\xi\) and \(\tau\) to \(\zeta = \xi - U_0 \tau\) and \(\tau = \tau\), where \(U_0\) is a constant velocity normalized by \(C_1\), and imposing the appropriate boundary conditions, viz.

\[
\psi^{(1)} \rightarrow 0, \quad \frac{\partial \psi^{(1)}}{\partial \zeta} \rightarrow 0 \quad \text{at} \quad \zeta \rightarrow \infty.
\]

Thus, one can express the stationary solution of the Burgers’ equation as [9]

\[
\psi^{(1)} = \psi_m \left[ 1 - \tanh \left( \frac{\zeta}{\Delta} \right) \right].
\]  

(37)
where the amplitude $\psi_m^{(1)}$ (normalized by $k_B T_i/\epsilon$) and the width $\Delta$ (normalized by $\lambda_D$) are given by

$$\psi_m^{(1)} = \frac{U_0}{A}, \quad (38)$$

$$\Delta = \frac{2C}{U_0}. \quad (39)$$

It is observed from Eqs. (37)–(39) that the amplitude (width) of the shock waves increases (decreases) as $U_0$ increases (decreases). It is also clear from Eq. (37) that shock potential profile is positive (negative) when $A$ is positive (negative).

Figure 1 exhibits the effect of spectral index $\kappa$ on the propagation of DA shock waves. It is obvious from figure that for the small changes in spectral index $\kappa$ exhibits different shock wave structures. The shock wave behavior can be seen from the above figure with positive potential. Later we are showing that $\kappa$ changes the amplitude of the DA shock waves significantly.

Figure 2 describes the structure of the positive DA shock wave profiles for different values of $\mu_e$. It is seen from the above figure that DA shock wave potential increases as $\mu_e$ increases.

It is seen from Figure 3 that for $\beta = 400, 500, 650$, the DA shock structure becomes different. It is clear from the above figure that positive DA shock wave potential decreases as $\beta$ increases. We also find that for various small values of the parameter $\alpha$, the DA shock structure becomes different (not shown in figure).

Figure 1. Variation of the DA shock wave potential $\psi^{(1)}$ with spatial coordinate $\zeta$ for different values of spectral index $\kappa$, where $\alpha = 0.1, \beta = 500, \sigma = 0.5, \mu_i = 0.5, \mu_e = 0.3, \eta_{10} = 0.1, \eta_{20} = 0.1, V_0 = 10, \kappa = 0.8$ (solid line), $\kappa = 1.0$ (dotted line), and $\kappa = 1.5$ (dashed line).
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Figure 2. Variation of the DA shock wave potential $\psi^{(1)}$ with spatial coordinate $\zeta$ for different values of $\mu_e$, where $\kappa = 1.1$, $V_0 = 10$ and the other parameters’ values are the same as in Figure 1, $\mu_e = 0.3$ (solid line), $\mu_e = 0.4$ (dotted line), and $\mu_e = 0.5$ (dashed line).

Figure 3. Variation of the DA shock wave potential $\psi^{(1)}$ with spatial coordinate $\zeta$ for different values of $\beta$, where $\kappa = 1.1$ and the other parameters’ values are the same as in Figure 1, $\beta = 400$ (solid line), $\beta = 500$ (dotted line), and $\beta = 650$ (dashed line).

In Figure 4, the variation of the amplitude of the DA shock waves is plotted against spectral index $\kappa$ for different values of the parameter $\sigma$ (ratio of ion temperature to electron temperature). It is seen from the above figure that when $\sigma$ increases, enhances the amplitude of the DA shock waves structure.

In Figure 5, the variation of the amplitude of the DA shock waves is plotted against spectral index $\kappa$ for several values of parameter $\mu_e$. From the above figure it is clear that an increase in $\mu_e$ decreases the amplitude of the positive
DA shock potential structures. Here the range of spectral index $\kappa$ lies between 1 to 5 (i.e. $1 < \kappa < 5$). It is also seen from the figure that positive DA shock potential structure rapidly increases when values of the spectral index $\kappa$ is small (i.e. $\kappa < 2$), then DA shock wave potential becomes steady in nature.

Figure 6 shows the variation of the width ($\Delta$) as a function of the parameter $\mu_e$ for different values of parameter $\alpha$. It can be clearly seen from the above figure that the width of the DA shock waves decreases as $\mu_e$ increases, and the higher values of $\alpha$ gives shorter width of the shock waves.
We have studied the nonlinear propagation DA shock waves in an unmagnetized dusty plasma comprising of mobile positively and negatively charged dust particles, superthermal electrons, and Boltzmann distributed ions. The propagation of the small amplitude nonlinear DA shock waves in the unmagnetized dusty plasma system is considered by analyzing the solution of Burgers’ equation. Burgers’ equation is derived by using the standard reductive perturbation method. A detailed numerical analysis of the amplitude, width of the DA shock waves is performed by varying different important plasma parameters. It is seen that for the small values of viscosity coefficient of both negative (or positive) dust fluid, the spectral index $\kappa$ and the other parameters $\alpha$, $\beta$, $\mu_e$ and $\sigma$ play significant role in the formation of DA shock waves and existence of shock structures in this unmagnetized plasma system. It would be possible that positive DA shock potential may trap negatively charged dust particles which can attract dust particles of opposite polarity to form larger sized dust or to be coagulated into extremely large sized neutral dust in solar wind, interstellar medium, auroral zone, pulsar’s magnetosphere or even in laboratory experiments.

In conclusion, we have shown here how the basic features of the nonlinear DA shock waves are modified by the presence of positive, negative dust particles and the superthermal nature of the kappa distributed electrons. The results, which

5 Conclusions

The parameters chosen in our numerical calculations are completely relevant to different regions of space, viz. solar wind, interstellar medium, auroral zone [24,25]; pulsar’s magnetosphere [26], etc.
have been obtained from this investigation, would be useful in understanding the properties of DA shock waves in laboratory and in space dusty plasmas. The present results may be useful for understanding the existence of nonlinear potential structures that are observed in different regions of space (viz. solar wind, interstellar medium, auroral zone [24,25]). The present investigation may be also helpful in the understanding of the nonlinear propagation of waves in the pulsar’s magnetosphere [26].

References