The Effect of Soft-Edged Absorption Damping on ATIR

H. Stoyanov
Department of Quantum Electronics, Faculty of Physics, University of Sofia, 5, James Bourchier Blvd, Sofia 1164, Bulgaria

Received 18 May 2011
Revised 22 October 2011

Abstract. The recently proposed [1] simple interferometric method was applied in the study of a perfect three-layered system, formed by a glass prism and a metal attenuator. The gap formed by the glass and metal surfaces forms a plane parallel micro cuvette. In our case this cuvette is filled symmetrically with two liquids with Fermi distribution of the extinction coefficient (the imaginary part of the complex index of refraction) in the region of their mutual solution. In this manner an almost stepwise variation of the optical thickness of the separation is introduced. The specific distribution of the phase shift between the $p$- and $s$-components of the reflected field was deduced from the output signal of the shearing polarization interferometer. An experiment with a glass prism-liquid-plane silicon attenuator is described. The results are compared with the theoretical predictions. For angles of incidence larger than the critical one a good agreement between theory and experiment was observed.

PACS codes: 42.25.Fx, 42.25.Gy, 42.60.Jf

1 Introduction

The attenuated total internal reflection (ATIR) is an area of constant interest. It was initially used for creating fixed or variable phase retarders [2-4] and variable filters [5], in the optical refractometry of absorbing liquids [6-9], metals, biotissue [10], for study of the subwavelength surface relief [11], etc. A very important area of application of ATIR is the surface plasmon resonance effects [12] and their applications in material science [13]. The classical ATIR configuration is a perfect three-layered system with two parallel plane boundaries. All mediae are expected to be homogeneous. On the contrary any deviation from this model leads to nontraditional effects.
In the present paper we study the role of an isothermal diffusion leading to a Fermi-law distribution of the optical absorption on the reflected field. The recently described method [14] based on the reduced Rayleigh integral equation allows us to study the ATIR reflected field in the case of any form of the optical absorption. Nevertheless in the present work we demonstrate that for a system composed of almost plane boundaries the simpler Mueller matrix method gives sufficiently good results and also presents comparison between theory and experiment.

2 Optical Set-up

The optical system used in the experiment is shown in Figure 1 [8]. The He-Ne laser $L$ emits a linearly polarized beam which is expanded by the collimator $K$. The collimated beam propagates through the TIR prism $PR$. The glass prism and the attenuator are mounted on a precise rotating stage. The Rochon beamsplitter $R$ followed by the linear polarizer $Pol$ form a simple shearing interferometer. The interferometric fringes are detected by a linear array image sensor $Det$ and analyzed by a PC-AT compatible computer. All elements are mounted on a vibration isolated optical table. The fringe contrast and topology are the parameters of our study.

![Figure 1. Sketch of the equipment: $L$ – laser source; $K$ – collimator; $PR$ – glass prism; $MP$ – metal plate; $R$ – Rochon prism; $Pol$ – linear polarizer; $Det$ – photodiode array detector; $PC$ – personal computer with a framegraber.](image)
Both components of the incident field ($p$- and $s$-) undergo independent amplitude losses and phase shifts in the process of TIR [15]. They are analyzed by a common path polarization shearing interferometer. The coordinate system is oriented so that the $z$ axis is along the direction of propagation of the laser beam and the $x$ axis is in the plane of incidence. The field vector $E$ in the entrance pupil of the optical system subtends an azimuthal angle $\varepsilon$ with the plane of incidence. In the exit pupil of the electric field $E'$, represented by its Jones vector, is given in [15]

$$
\begin{bmatrix}
E'_p \\
E'_s
\end{bmatrix}
= 
\begin{bmatrix}
r_p & 0 \\
0 & r_s
\end{bmatrix}
\begin{bmatrix}
E_p \\
E_s
\end{bmatrix},
$$

(1)

where $E_p = |E| \cos \varepsilon$ and $E_s = |E| \sin \varepsilon$ are the components of the entrance Jones vector. The complex amplitude reflection coefficients are written as

$$
r_p = \rho_p \exp(i\delta_p), \quad r_s = \rho_s \exp(i\delta_s).
$$

(2)

The phase difference between the two components we denote by $\Delta = \delta_p - \delta_s$. This quantity can be determined out by analyzing the fringe pattern at the exit of the interferometer. The expressions for calculating $r_p$ and $r_s$ derived on the basis of the theory of stratified optical media are [15,16]

$$
r_s = \frac{Z_{0a}^2(A - CZ^f_{0a}) + (B - AZ^f_{0a})}{Z_{0a}^2(A + CZ^f_{0a}) + (B + AZ^f_{0a})},
$$

(3)

$$
r_p = \frac{Z_{0p}^2(CZ^f_{0p} - A) + (AZ^f_{0p} - B)}{Z_{0p}^2(CZ^f_{0p} + A) + (AZ^f_{0p} + B)}.
$$

(4)

The indices $i$ and $f$ refer to the initial and final media, respectively and $A, B, C$ are the elements of the transmission matrix through the gap with thickness $h$

$$
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
= \begin{bmatrix}
\cosh \alpha_2 h & Z_{0,2} \sinh \alpha_2 h \\
Z_{0,2}^{-1} \sinh \alpha_2 h & \cosh \alpha_2 h
\end{bmatrix}.
$$

(5)

Here we denoted the tangential wave vector component in the second medium by

$$
\alpha_2 = k \left[ \left( \frac{n_3}{n_2} \right)^2 \sin^2 \theta_3 - 1 \right]^{1/2}
$$

and the characteristic boundary impedance $Z_{0,2}$ by

$$
Z_{0,2} = (n_2 \cos \theta_2)^{-1} \quad \text{for } s\text{-polarization}
$$

(6a)

and

$$
Z_{0,2} = n_2^{-1} \cos \theta_2 \quad \text{for } p\text{-polarization},
$$

(6b)
The Effect of Soft-Edged Absorption Damping on ATIR

where
\[
\cos \theta_2 = -i \left( \frac{n_3}{n_2} \right)^2 \sin^2 \theta_3 - 1 \right)^{1/2}
\]
gives the direction of propagation of the wave in the second medium (in our case liquid). The characteristic boundary impedances of the initial and final media for both\(p\)- and\(s\)-component of the field are given respectively by
\[
Z_{0,s}^i = [(n_1 - i\kappa_1) \cos \theta_1]^{-1}, \quad Z_{0,s}^f = (n_3 \cos \theta_3)^{-1}, \quad \text{(7)}
\]
\[
Z_{0,p}^i = (n_1 - i\kappa_1)^{-1} \cos \theta_1, \quad Z_{0,p}^f = n_3^{-1} \cos \theta_3, \quad \text{(8)}
\]
where \(\sin \theta_1 = (n_3 \sin \theta_3)/(n_1 - i\kappa_1)\), \(\cos \theta_1 = (1 - \sin^2 \theta_1)^{1/2}\), \(\Re[\cos \theta_1] > 0\), \(n_3\) is the index of refraction of the glass prism \(PR\); \(\hat{n}_1 = n_1 - i\kappa_1\) is the complex index of refraction of the bulk absorbing medium \(MP\). For angle of incidence \(\theta_3\) slightly exceeding the critical angle of TIR, the complex amplitude coefficients (2) exhibit strong dependence on the boundary conditions. These conditions are functions of the optical properties of all media. We suppose constant and homogeneous optical properties of the glass prism and the metal attenuator. Two drops, one of the dye (malachite green) and second of water were added closely on both sides of the center of the prism hypotenuse face (Figure 2). A short time later the liquids contact and a mutual diffusion starts.

Here we suppose a quasistatic dynamics of the mixture what means that the wavelength of the optical field is much larger than the inclusion diameter. The particles of dye in solvent are spheres of permittivity \(\varepsilon_i\). They are randomly

Figure 2. The geometry of the micro cuvette and the \(n_2(x)\) distribution.
spread in a homogeneous environment $\varepsilon_e$ and occupy a volume fraction $f$. According to the Maxwell-Garnett mixing rule the mixture will have effective permittivity $\varepsilon_{\text{eff}}$ given in [17,18]

$$
\varepsilon_{\text{eff}} = \varepsilon_e + 2f\varepsilon_e\frac{\varepsilon_i - \varepsilon_e}{\varepsilon_i + \varepsilon_e - f(\varepsilon_i - \varepsilon_e)}
$$

and the characteristic boundary impedances in the second medium which will take part in (6a,b) will be

$$
Z = Z_0 \frac{1 + i\varepsilon_{\text{eff}}^{-1/2}\tan(kw)}{1 + i\varepsilon_{\text{eff}}^{1/2}\tan(kw)},
$$

where $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ is the free space impedance, $k = (\omega/c_0)\varepsilon_{\text{eff}}^{1/2}$ is the wave number, $\omega$ is the angular frequency and $c_0$ is the velocity of light in a vacuum, $w$ is the length of the irradiated area. From the Fick’s second law we can assume that the index of refraction can be expressed by a Fermi function (Figure 2) in the form

$$
n_2(x) = \sqrt{\varepsilon_{\text{eff}}} = n_0 + \frac{1}{\exp[\alpha(x - x_0)] + 1} \cdot N,
$$

where $n_0$ and $N$ are real constants, $\varepsilon_{\text{eff}}$ is the effective permeability (9) of the liquid solution. Then the reflected field will be mainly dependent on the angle of incidence $\theta_3$ and on the optical thickness $n_2(x)h$ of the gap.

If the detector (Figure 1) is a linear photodiode array oriented along the $x$ axis, the measured fringe intensity distribution is [1]

$$
I(x) = I_0 \cos^2(\varepsilon) \cos^2(\gamma) \left\{ \rho_p^2 + \rho_s^2 \tan^2(\varepsilon) \tan^2(\gamma) + 2\rho_p\rho_s \tan(\varepsilon) \tan(\gamma) \cos[\Phi(x, \Delta)] \right\}.
$$

The phase term $\Phi(x, \Delta)$ has the form

$$
\Phi(x, \Delta) = (k_p - k_s) \cdot r + \frac{2\pi}{\lambda} d(x) \left[ n_0 - \frac{n_e(\beta)}{\cos(\beta)} \right] + \Delta,
$$

where $\Delta = \delta_p - \delta_s$ is the phase difference between both components, $\beta$ is the angle deviation between the ordinary and the extraordinary rays at the exit of the Rochon prism (the angular shear). Figure 2, $\gamma$ is the azimuth of the polarizer, measured from the plane of incidence $O, x, z$, $d(x)$ is the current thickness of the second right angle prism of the Rochon prism, $r$ is the radius vector of the current point of measurement. In our case it lays in the plane of incidence $O, x, z$, $k_p, k_s$ are the wave vectors of the ordinary and extraordinary waves, which are corresponding to the $p$- and $s$-components of the ATIR field, respectively.

The first two terms in equation (11) form the carrier frequency of the interferometric signal. For a plane wave this signal represents a family of straight
parallel fringes. The third term ($\Delta = \delta_p - \delta_s$) in (11), which is a function of additional physical and geometrical parameters, leads to distortion of the basic set of fringes provided that the azimuthal angles $\varepsilon$ and $\gamma$ are kept constant.

From the expressions for $r_p$ and $r_s$ it is clear that $\rho_p$, $\rho_s$ and $\Delta$ depend on the complex optical thickness $\tilde{n}_2(x)h$ of the micro cuvette (the gap), on the angle of incidence and on the optical properties of the media the light is passing through. The theoretical investigation of the $h$-dependence of $\rho_p$, $\rho_s$ [1] shows that it is not as strong and can be neglected in the first approximation. The interferometric signal, detected by the linear detector, is governed mainly by the phase difference $\Delta$. The numerical model of the Eq. (9) allows predicting the intensity distribution in the output interferometric signal along the axes of the linear sensor for various optical materials and for any polarization state of the incident wave. Even the case of non homogeneous intermediate medium can be studied in this way. In Figure 3 it is shown the theoretical fringe intensity distribution along the aperture of the linear sensor as a function of the separation $h$ (the thickness of the micro cuvette) between the TIR prism and the metal attenuator for the media used in the following experiment, silicon mono-crystal $\tilde{n}_1 = 3.83 - i0.02$ [19], liquid $n_2(x)$, $n_3 = 1.56687$ Schott BaK4 glass.

3 Experimental Results and Discussions

The source of linearly polarized coherent light in Figure 1 is a HeNe laser (Melles Griot, 3 mW), working in TEM$_{00}$ mode. The beam is expanded and collimated up to approximately 50 mm dia. by a well corrected collimator (Jodon, model BET-50). From the so generated plane wave only a small part was used — approximately $2 \times 15$ mm in the central area of the aperture. The prism $\text{PR}$ was made of optical glass BaK4 (Schott Jena) with $n_3 = 1.56687$ at $\lambda = 632.8$ nm. The polarization interferometer consists of a Rochon prism.
and a sheet linear polarizer. The Rochon prism provides an angular shear $\beta$ of about $1.77 \times 10^{-4}$ [rad], so that only approximately four fringes cover the full aperture of the sensor. The photodiode array image sensor (Matsushita, model MN–512K) with 512 elements was used to detect photometric sections trough the interferometric fringes. The sensor pixel dimensions are $28 \times 16 \, \mu m$ and the pitch is $28 \, \mu m$. The total length of the sensitive area is 14.3 mm. The ADC converter of the slot card framegraber provides 8 bits quantization of the video signal. Special attention has been paid to the determination of the gap thickness $h$. This measurement was carried out visually (in a manner similar to the described in [1]) by means of measuring microscope (not shown in Figure 1) with long working distance lens (about 122 mm). The white light Fizeau fringes were observed in direction approximately normal to the hypotenuse of the TIR prism. The tilt adjustment was accomplished by a tree point micro screw adjusting stage mounted on the rotating table. From the initial situation the sample was translated step by step away from the prism and the movement the dark brown-yellow fringe ($\lambda = 419 \, nm$ approximately) was registered by the microscope. The wedge angle value between the prism hypotenuse and the metal surface was approximately $0.5 \times 10^{-4}$ [rad]. The angle of incidence was measured and adjusted by a precise rotating stage (Thorlabs, Model 7R7) which offers resolution of 1 arc minute. An optically polished plate of silicon mono-crystal, N-type, cut at $(1, 0, 0)$ ($\hat{n}_1 = 3.85 - i0.02$ at $\lambda = 632.8 \, nm$ [19]) was used as attenuator.
The Effect of Soft-Edged Absorption Damping on ATIR

The object of our experiment was the intermediate space initially filled with air. Two tiny drops, one of the dye and second of water, were added closely to both sides of the center of the prism hypotenuse face. A short time later the liquids contact and a mutual diffusion starts. The array sensor allows us to capture and register almost linear photometric sections of the fringe intensity distribution. The signal shown in Figure 4 was captured approximately 7 seconds later. For angles of incidence exceeding the critical one the interferometer forms approximately four full fringes in the input pupil of the sensor. As have been expected the presence of symmetrical distribution of the absorption leads to deformation and slight side shift of the fringes together with decreasing of the fringe visibility in the reflected field. The experimental results shown in Figure 4 are in a quite good agreement with the numerically predicted intensity distribution for the situations when the intermediate space is filled with air.

4 Conclusion

The method here described proved to be a successful and versatile tool for ATIR investigation of absorbing liquid medium. The experiments gave results in a quite good agreement with the theory. The method can be used not only in the research, but in other applications connected with the sensor techniques like contamination monitoring, refractometry of absorbing media, etc.

Acknowledgments

This work was supported by the National Science Fund of Bulgaria, grant No. DO-02-0114-2008 and grant No. DRNF-02/8-2009.

References

H. Stoyanov


