Interacting Cosmic Fluids in LRS Bianchi Type-I Cosmological Models

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Abstract. We have studied anisotropic, homogeneous LRS Bianchi type I two-fluid cosmological models dominated by two interacting perfect fluid components. To get determinate solutions of the field equations we have assumed a barotropic equation of state for pressure and density. It is shown that due to the mutual interaction between the two fluids, the energy densities are proportional to $1/t^2$.

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1 Introduction

In the study of modern cosmology, we consider that the total energy density of the universe is dominated by the densities of two components: the dark matter and the dark energy [1]. The recent observational data strongly motivate to study general properties of cosmological models containing more than one fluid. These universes are modeled with perfect fluids and with mixtures of non-interacting fluids [2] under the assumption that there is no energy transfer among the components. But, such scenarios are not confirmed by observational data. This motivates us to study cosmological models containing fluids which interact with each other.

The cosmological models with energy transfer among these fluids explained the late time acceleration of the universe [3] and the coincidence problem [4,5]. Tolman [6] and Davidson [7] have considered the interaction between dust-like matter and radiation. Gromov et al. [8] have studied cosmological models with decay of massive particles into radiation or with matter creation. Cataldo et al. [9] have considered the simplest non-trivial cosmological scenarios for an interacting mixture of two cosmic fluids described by power-law scale factors, i.e. the expansion (contraction) as a power-law in time. An interacting two-fluid scenario for dark energy in an FRW universe have been studied by Amirhashchi
et al. [10]. Whereas, an interacting and non-interacting two-fluid scenario for
dark energy in an FRW universe with constant deceleration parameter has been
recently described by Pradhan et al. [11].

Generally, the power law cosmologies are defined by the growth of the cosmic-
ological scale factor as $a(t) = t^\alpha$. For $\alpha > 0$ the universe is expanding, whereas
for $\alpha < 0$ we have contracting universe ($t > 0$). The Hubble parameter $H_0 = \dot{a}/a$
and the deceleration parameter $q(t) = -\ddot{a}/\dot{a}^2$ completely describe the behav-
ior of the universe in power law cosmologies. For $a(t) = t^\alpha$ it reduces to the
form $q(t) = -(\alpha - 1)/\alpha$, which implies that the universe expands with a con-
stant velocity for $\alpha = 1$. The universe expands with an accelerated expansion
for $\alpha > 1$, since, if the expansion is speeding up, the deceleration parameter
must be negative.

A simple inflationary model characterized by a period in which the scale factor
is a power law in time with $\alpha > 1$ can also be considered [12].

There is a class of cosmological models which dynamically solve the cosmic-
ological constant problem, where the scale factor grows as a power law in time
regardless of the matter content.

In this study, it is shown that the mutual exchange of energy between two perfect
fluids can be described by energy densities which are proportional to $1/t^2$ and
the interacting term proportional to $1/t^3$. Due to consideration of interacting
fluids, we get the energy densities evolve at the same rate and their ratio is con-
stant. This satisfies the so called cosmological coincidence problem. The aim
of this paper is to consider the simplest non-trivial cosmological scenario for an
interacting mixture of two cosmic fluids described by power law scale factors.

2 Field Equations

We consider LRS Bianchi type-I universe as

$$ds^2 = -dt^2 + A^2 dx^2 + B^2(dy^2 + dz^2),$$

(1)

where $A$ and $B$ are functions of cosmic time $t$.

The Einstein field equations are

$$R_{ij} - \frac{1}{2} g_{ij} R = -T_{ij},$$

(2)

where $R_{ij}$ is the Ricci tensor and $R$ is the Ricci scalar.

The energy momentum tensor is

$$T_i^j = (\rho + p)u_i u^j - p\delta_i^j$$

with components

$$T_1^1 = T_2^2 = T_3^3 = -p_1 - p_2, \quad T_4^4 = \rho_1 + \rho_2,$$

(3)
where $p_1$, $p_2$ and $\rho_1$, $\rho_2$ are pressures and densities of two fluids satisfying barotropic equations of state

\[
\begin{align*}
p_1 &= \omega_1 \rho_1 \\
p_2 &= \omega_2 \rho_2
\end{align*}
\]

with $\omega_1$ and $\omega_2$ constant state parameters.

The field equations (2) with equations (1) and (3) reduce to

\[
\begin{align*}
2 \dot{H}_2 + 3 H_2^2 &= p_1 + p_2, \\
\dot{H}_1 + \dot{H}_2 + H_1^2 + H_2^2 + H_1 H_2 &= p_1 + p_2, \\
H_2^2 + 2 H_1 H_2 &= -(\rho_1 + \rho_2). 
\end{align*}
\]

(4)

Here $H_1 = \dot{A}/A$ and $H_2 = \dot{B}/B$ are the directional Hubble parameters.

So that, the mean Hubble parameter is $H = \frac{1}{3}(H_1 + 2 H_2)$ (see Tripathi [13]).

We take a linear relationship between the Hubble parameters $H_1$ and $H_2$ as

\[
H_1 = k H_2,
\]

(5)

where $k$ is the arbitrary constant usually assumed to take positive values only and it takes care of anisotropic nature of the model.

Using (5), the above field equations reduce to

\[
\begin{align*}
2 \dot{H}_2 + 3 H_2^2 &= p_1 + p_2, \\
(k + 1) \dot{H}_2 + (k^2 + k + 1) H_2^2 &= p_1 + p_2, \\
(2k + 1) H_2^2 &= -(\rho_1 + \rho_2).
\end{align*}
\]

(6) \quad (7) \quad (8)

The conservation law $T^i_{;j} = 0$ implies that the two components $\rho_1$ and $\rho_2$ interact through the interacting term $Q$ as follows:

\[
\begin{align*}
\dot{\rho}_1 + 3H(\rho_1 + p_1) &= Q, \\
\dot{\rho}_2 + 3H(\rho_2 + p_2) &= -Q.
\end{align*}
\]

(9) \quad (10)

The nature of the interacting term $Q$ is not clear at all.

If $Q > 0$, then there exists an energy transfer from the fluid $\rho_2$ to the fluid $\rho_1$.

If $Q = 0$, then we have two non-interacting fluids, each satisfying the standard conservation equation separately.

We consider different forms of the interacting term $Q$ for solving these equations.

By putting $Q = 0$ in the equations (9) and (10), we get

\[
\rho_1 = m_1 (AB^2)^{-1(w_1)} \quad \text{and} \quad \rho_2 = m_2 (AB^2)^{-1(w_2)},
\]

where $m_1$ and $m_2$ are constants of integration.
3 Solutions of the Field Equations

Consider interacting matter sources satisfying barotropic equation of state

\[ p_1 = w_1 \rho_1, \quad p_2 = w_2 \rho_2, \]

where \( w_1 \) and \( w_2 \) are constant state parameters.

We define the scale factor as \( AB^2 = t^\alpha \), where \( \alpha \) is a constant parameter.

From this, we get \( H = \alpha/t \).

If we consider \( \alpha = 1 \), then we get \( AB^2 = t \), i.e., there is no acceleration and the universe will expand or collapse with constant velocity.

From the resultant of equations (9) and (10), we obtain

\[ \rho_{k1} = \frac{3\alpha(2k + 1)[2 - 3\alpha(1 + w_2)]}{(k + 2)^2(w_2 - w_1)t^2}, \quad (11) \]

\[ \rho_{k2} = \frac{3\alpha(2k + 1)[3\alpha(1 + w_1) - 2]}{(k + 2)^2(w_2 - w_1)t^2}, \quad (12) \]

where \( \rho_{k1} \) and \( \rho_{k2} \) are densities of first and second fluid respectively.

And the interacting term is given by

\[ Q = \frac{3\alpha(2k + 1)[2 - 3\alpha(1 + w_2)][3\alpha(1 + w_1) - 2]}{(k + 2)^2(w_2 - w_1)t^3}. \quad (13) \]

From equations (6) and (7), we get

\[ p_1 = \frac{3\alpha}{2(k + 2)^2(w_2 - w_1)t^2} \left\{ 3\alpha(k^2 + k + 4) - (k^2 + 5k + 6)[w_2 - w_1] - 2(2k + 1)w_2[3\alpha(1 + w_1) - 2] \right\} \]

\[ p_2 = \frac{3\alpha}{2(k + 2)^2(w_2 - w_1)t^2} \left\{ 3\alpha(k^2 + k + 4) - (k^2 + 5k + 6)[w_2 - w_1] - 2(2k + 1)w_1[2 - 3\alpha(1 + w_2)] \right\}. \quad (14) \]

We consider non-zero values of \( k \) (i.e. \( k \neq 0 \)).

For \( k = -\frac{1}{2} \), the equations (11) and (12) imply that the energy densities \( \rho_{k1} \) and \( \rho_{k2} \) vanish. In this case, the cosmological evolution with two interacting fluids is not possible.

If we consider \( k = 1 \), then the space time get converted to FRW and results for it are already proved by Cataldo [9].
Interacting Cosmic Fluids in LRS Bianchi Type-I Cosmological Models

If

\[ w_1 \geq -\frac{1}{3} \quad \text{and} \quad w_2 \leq -\frac{1}{3}, \quad \text{for} \quad w_1 > w_2, \quad (16) \]

then we get \( \rho_{k1} \geq 0 \) and \( \rho_{k2} \geq 0 \).

From equation (16), we conclude that always one of the interacting fluids must be either a dark fluid or a phantom fluid.

The constraints (16) on the state parameter using equation (13) imply that \( Q < 0 \). Therefore, the energy is transferred from a dark fluid \((-1 \leq \omega_1 \leq -\frac{1}{3})\) or a phantom fluid \((\omega_1 < -1)\) to the another matter component whose state parameter is \( \omega_2 > -\frac{1}{3} \).

Also, we consider the behavior of the constant ratio \( r \) of energy densities given as

\[ r = \frac{\rho_{k2}}{\rho_{k1}} = -\frac{1 + 3w_1}{1 + 3w_2}. \]

This is a function of the model parameters \( w_1 \) and \( w_2 \).

This ratio is always constant because the energy densities are proportional to \((1/t^2)\).

Now we consider two specific fluid interactions.

**[i] Dust–Perfect Fluid Interaction** \((w_1 = 0, w_2 \neq 0)\)

Here, we consider the interaction of dust with any other perfect fluid. So we have \( w_1 = 0 \) and \( w_2 \) is a free parameter.

We have for a dust \((d)\) and a perfect fluid \((pf)\) interacting configurations with the equations of state \( p_d = 0, p_{pf} = \omega_2 \rho_{pf} \), so we get

\[ \rho_d = \frac{3\alpha(2k+1)[2-3\alpha(1+w_2)]}{(k+2)^2w_2t^2}, \quad (17) \]

\[ \rho_{pf} = \frac{3\alpha(2k+1)[3\alpha-2]}{(k+2)^2w_2t^2}, \quad (18) \]

\[ p_d = \frac{3\alpha\{3\alpha(k^2+k+4)-(k^2+5k+6)w_2-2(2k+1)w_2[3\alpha-2]\}}{2(k+2)^2w_2t^2}, \quad (19) \]

\[ p_{pf} = \frac{3\alpha\{3\alpha(k^2+k+4)-(k^2+5k+6)w_2\}}{2(k+2)^2w_2t^2}, \quad (20) \]

\[ Q = \frac{3\alpha(2k+1)[2-3\alpha(1+w_2)][3\alpha-2]}{(k+2)^2w_2t^3}. \quad (21) \]

For the requirement of simultaneous fulfillment of the conditions \( \rho_1 \geq 0 \) and \( \rho_2 \geq 0 \) we must have

\[ \frac{2}{3} < \alpha < \frac{2}{3(1+w_2)} \quad (w_2 > 0). \]

We consider an example of the dust-radiation interaction.
So, we specifically put \( w_1 = 0, w_2 = \frac{1}{3} \), and we get

\[
\rho_d = \frac{9\alpha(2k+1)[2-4\alpha]}{(k+2)^2t^2},
\]

(22)

\[
\rho_{pf} = \frac{9\alpha(2k+1)[3\alpha-2]}{(k+2)^2t^2},
\]

(23)

\[
p_d = \frac{3\alpha\left\{3\alpha(k^2+k+4)-(k^2+5k+6)\right\}-2(2k+1)[3\alpha-2]}{2(k+2)^2t^2} = 0 \text{ for dust},
\]

(24)

\[
p_{pf} = \frac{3\alpha[3\alpha(k^2+k+4)-(k^2+5k+6)]}{2(k+2)^2t^2},
\]

(25)

\[
Q = \frac{9\alpha(2k+1)[2-4\alpha][3\alpha-2]}{(k+2)^2t^2}.
\]

(26)

For \( k = 1 \) and \( k = 2 \) the pressure of dust is zero. Also the barotropic equation of state \( p_{pf} = w_2\rho_{pf} \) is satisfied.

We get positive energy densities if \( \alpha < \frac{1}{2} \) or \( \alpha > \frac{2}{3} \).

The interaction term \( Q \) is also positive for the same interval. This means that we have transfer of energy from radiation to dust. In another words, there exists a dust-radiation interacting LRS Bianchi type-I cosmological model dominated by radiation (for \( \alpha > \frac{2}{3} \)) or dominated by dust (for \( \alpha < \frac{1}{2} \)) throughout all their evolution.

[ii] Phantom Fluid- Perfect Fluid Interaction \( (w_1 = -\frac{4}{3}, w_2 \neq 0) \)

We consider the interaction of a phantom fluid with any other perfect fluid configuration. The equations of states are

\[
p_{ph} = -\frac{4}{3}p_{ph}, \quad p_{pf} = \omega_2\rho_{pf}.
\]

Here \( \omega_2 \) is a free parameter. Thus, we have for a phantom fluid (ph) and a perfect fluid (pf) interacting configuration as

\[
\rho_{ph} = \frac{9\alpha(2k+1)[2-3\alpha(1+w_2)]}{(k+2)^2(3w_2+4)t^2},
\]

(27)

\[
\rho_{pf} = \frac{9\alpha(2k+1)[-\alpha-2]}{(k+2)^2(3w_2+4)t^2},
\]

(28)

\[
p_{ph} = \frac{3\alpha\left\{3\alpha(k^2+k+4)-(k^2+5k+6)[3w_2+4]-2(2k+1)w_2[3\alpha+6]\right\}}{2(k+2)^2(3w_2+4)t^2},
\]

(29)

\[
p_{pf} = \frac{3\alpha\left\{3\alpha(k^2+k+4)-(k^2+5k+6)(3w_2+4)+8(2k+1)[2-3\alpha(1+w_2)]\right\}}{2(k+2)^2(3w_2+4)t^2}.
\]

(30)
Interacting Cosmic Fluids in LRS Bianchi Type-I Cosmological Models

\[ Q = \frac{9\alpha(2k+1)[2-3\alpha(1+w_2)][-\alpha-2]}{(k+2)^2(3w_2+4)t^3}. \] (31)

In order to get the positive energy density for phantom fluid and perfect fluid, the following constraints must be satisfied:

\[ -2 < \alpha < \frac{2}{3(1+w_2)} \quad (w_2 > -1) \]
\[ \frac{2}{3(1+w_2)} < \alpha < -2 \quad (-4/3 < w_2 < -1) \]
\[ -2 < \alpha < \frac{2}{3(1+w_2)} \quad (w_2 < -4/3), \]

i.e., when phantom fluid interacts with perfect fluid, the universe expands only when \( w_2 > -1 \).

We consider an example of the interaction of above phantom fluid with dust distribution. In this case, we have \( w_{ph} = -4/3, w_d = 0 \).

For \( k = 1 \) and \( k = 2 \) the barotropic equation of state \( p_{ph} = -4\rho_{ph}/3 \) is satisfied.

The interacting term \( Q \) is positive (for \( \alpha > -2 \) and \( \alpha < 2/3 \)), so there is transfer of energy from dust to phantom matter.

4 The Effective Fluid Interpretation

In this Section we wish to associate an effective fluid interpretation with the interaction of the two fluid mixture. Therefore, we equate sum of pressures \( p_1 \) and \( p_2 \) with an effective pressure \( p \). We get

\[ p = p_1 + p_2, \quad \therefore \quad p = w_1\rho_1 + w_2\rho_2. \] (32)

We have the equation of state

\[ p = \gamma \rho = \gamma(\rho_1 + \rho_2), \] (33)

where \( \gamma \) is the constant effective state parameter.

We should note that the equation of the state of the associated effective fluid is not produced by physical particles and their interactions (refer Catldo et al. [9]).

The effective state parameter \( \gamma \) is related to the parameter \( \alpha \) by

\[ \gamma = \frac{(k^2 + k + 4) - 3\alpha(k^2 - k + 3)}{3\alpha(2k + 1)} \] (34)

Hence, for all positive values of \( k \), we note that the effective state parameter \( \gamma \rightarrow -1 \) as \( \alpha \rightarrow \pm \infty \). Also, for \( \alpha < 0 \), we have the phantom sector (because \( \gamma < -1 \)).
5 Conclusion

In this paper we have discussed the behavior of LRS Bianchi type I power law scaling cosmological model dominated by two interacting perfect fluid components during the expansion.

We have mathematically shown that energy densities for both components are positive ($\rho_{k1} \geq 0$ and $\rho_{k2} \geq 0$) for $w_1 \geq -\frac{1}{3}$ and $w_2 \leq -\frac{1}{3}$, where $w_1 > w_2$. There exist dust-radiation interacting cosmological scenarios dominated by dust or radiation throughout all evolution when $\alpha < \frac{1}{2}$ or $\alpha > \frac{2}{3}$.

Energy transfer takes place from dust to the phantom fluid when $\alpha > -2$ aAnd $\alpha < \frac{2}{3}$.

The power law inflationary model can be filled by two interacting fluids with state parameter given by $w_2 \leq -\frac{1}{3} \leq w_1$.

We have shown that energy densities of two interacting perfect fluid components are proportional to $(t^{-2})$ and the interaction term is proportional to $(t^{-3})$ in all cases.

Finally, we have investigated the effective fluid interaction in the model.

We have noted that the expression for the effective state parameter $\gamma$ is a function of $\alpha$.

References


378
Interacting Cosmic Fluids in LRS Bianchi Type-I Cosmological Models