LRS Bianchi Type V Universe in Creation-Field Cosmology

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Abstract. The Hoyle-Narlikar $C$-field cosmology with LRS Bianchi type-V space-time has been studied. Using methods of Narlikar and Padmanabham [1], the solutions have been obtained when the creation field $C$ is a function of time $t$ only. The geometrical and physical aspects of the model are also examined.

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1 Introduction

The phenomenon of expanding universe, primordial nucleon-synthesis and the observed isotropy of cosmic microwave background radiation (CMBR) are the three very important observations in astronomy. These were successfully explained by big-bang cosmology based on Einstein’s field equations. Smoot et al. [2] revealed that the earlier predictions of the Friedman-Robertson-Walker type of models do not always exactly match with our expectations. Some puzzling results regarding the red shifts from the extra galactic objects continue to contradict the theoretical explanations given from the big-bang type of the model. Also, CMBR discovery did not prove it to be a outcome of big-bang theory. In fact, Narlikar et al. [3] have proved the possibility of non-relic interpretation of CMBR. To explain such phenomenon, many alternative theories have been proposed from time to time. Hoyle [4], Bondi and Gold [5] proposed steady state theory in which the universe does not have singular beginning nor an end on the cosmic time scale. To overcome this difficulty Hoyle and Narlikar [6] adopted a field theoretic approach by introducing a massless and chargeless scalar field $C$ in the Einstein-Hilbert action to account for the matter creation. In the $C$-field theory introduced by Hoyle and Narlikar there is no big-bag type of singularity as in the steady state theory of Bondi and Gold [5]. A solution of Einstein’s field equations admitting radiation with negative energy massless scalar creation field $C$ was obtained by Narlikar and Padmanabhan [1].
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The study of Hoyle and Narlikar theory [6–8] to the space-times with higher dimensions was carried out by Chatterjee and Banerjee [9]. RajBali and Tikekar [10] studied $C$-field cosmology with variable $G$ in the flat Friedmann-Robertson-Walker model and with non-flat FRW space-time by RajBali and Kumawat [11]. The solutions of Einstein’s field equations in the presence of creation field have been obtained for different Bianchi type universes by Singh and Chaubey [12].

We have examined a LRS Bianchi type-V cosmological model in Hoyle and Narlikar $C$-field cosmology. We have assumed that the creation field $C$ is a function of time $t$ only, i.e. $C(x, t) = C(t)$.

2 Hoyle and Narlikar $C$-field Cosmology

Introducing a massless scalar field called as creation field viz. $C$-field, Einstein’s field equations were modified by Hoyle and Narlikar (1963, 1964 and 1966). The modified field equations are

$$R_{ij} - \frac{1}{2}g_{ij}R = -8\pi\left(mT_{ij} + cT_{ij}\right),$$

where $mT_{ij}$ is a matter tensor of Einstein’s theory and $cT_{ij}$ is a matter tensor due to the $C$-field which is given by

$$cT_{ij} = -f\left(C_iC_j - \frac{1}{2}g_{ij}C^kC_k\right),$$

where $f > 0$ and $C_i = \frac{\partial C}{\partial x^i}$.

Because of the negative value of $T^{00}$ ($T^{00} < 0$), the $C$-field has negative energy density producing repulsive gravitational field which causes the expansion of the universe. Hence, the energy conservation equation reduces to

$$mT_{ij} = -cT_{ij} = fC^iC^j,$$

i.e. the matter creation through non-zero left hand side is possible while conserving the over all energy and momentum.

The Above equation is similar to

$$mg_{ij}\frac{dx^i}{ds} - C_j = 0,$$

which implies that the 4-momentum of the created particle is compensated by the 4-momentum of the $C$-field. In order to maintain the balance, the $C$-field must have negative energy.
3 Metric and Field Equations

The LRS Bianchi Type-V line element can be written as
\[ ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 e^{-2mx} (dy^2 + dz^2) , \]  
(5)
where the scale factors \( a_1 \) and \( a_2 \) are functions of \( t \) only and \( m \) is a constant.

We have assumed that creation field \( C \) is a function of time \( t \) only, i.e.
\[ C(x, t) = C(t) \quad \text{and} \quad m T^i_j = \text{diag}(\rho, -p, -p, -p). \]  
(6)

We have assumed that the velocity of light and the gravitational constant are equal to one unit.

Now, the Hoyle-Narlikar field equations (1) for metric (5) with the help of equations (2), (3), and (6) will reduce to the following set of equations:

\[ \frac{\ddot{a}_1}{a_1 a_2} + \frac{\ddot{a}_2}{a_2} - \frac{3m^2}{a_1^2} = 8\pi \left( \rho - \frac{1}{2} f\dot{C}^2 \right) , \]  
(7)
\[ 2 \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_2^2}{a_2} - \frac{m^2}{a_1^2} = 8\pi \left( -p + \frac{1}{2} f\dot{C}^2 \right) , \]  
(8)
\[ \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} - \frac{m^2}{a_1} = 8\pi \left( -p + \frac{1}{2} f\dot{C}^2 \right) , \]  
(9)
\[ \frac{\dot{a}_1}{a_1} = \frac{\dot{a}_2}{a_2} , \]  
(10)
\[ \dot{\rho} + \left( \frac{\dot{a}_1}{a_1} + 2 \frac{\dot{a}_2}{a_2} \right) (\rho + p) = f\dot{C} \left[ \dot{C} + \left( \frac{\dot{a}_1}{a_1} + 2 \frac{\dot{a}_2}{a_2} \right) \dot{C} \right] , \]  
(11)
where dot (\( \cdot \)) indicates the derivative with respect to \( t \).

From equation (10), we get
\[ a_1 = ca_2 , \]  
(12)
where \( c \) is the constant of integration.

The spatial volume is given as
\[ V = a_1 a_2^2 . \]  
(13)
From equation (12) and equation (13), we get
\[ a_1 = c^{\frac{2}{3}} V^{\frac{1}{3}} . \]  
(14)

The above equation (11) can be written in the form
\[ \frac{d}{dV} (V \rho + p) = f\dot{C} (V) \frac{d}{dV} [V \dot{C} (V)] . \]  
(15)
In order to obtain a unique solution, one has to specify the rate of creation of matter-energy (at the expense of the negative energy of the C-field). Without loss of generality, we assume that the rate of creation of matter energy density is proportional to the strength of the existing C-field energy-density, i.e. the rate of creation of matter energy density per unit proper-volume is given by

\[
\frac{d}{dV} (V \rho) + p = \alpha^2 \dot{C}^2 \equiv \alpha^2 g^2(V),
\tag{16}
\]

where \( \alpha \) is a proportionality constant and we have defined \( \dot{C}(V) \equiv g(V) \).

Substituting it in equation (15), we get

\[
\frac{d}{dV} (V \rho) + p = f g(V) \frac{d}{dV} (V g).
\tag{17}
\]

Comparing right hand sides of equations (16) and (17), we get

\[
g(V) \frac{d}{dV} (gV) = \frac{\alpha^2}{f} g^2(V).
\tag{18}
\]

Integrating, which gives

\[
g(V) = A_1 V^{(\alpha^2/f - 1)},
\tag{19}
\]

where \( A_1 \) is arbitrary constant of integration.

We consider the equation of state of matter as

\[p = \gamma \rho.\tag{20}\]

Substituting equations (19) and (20) in the equation (16), we get

\[
\frac{d}{dV} (V \rho) + \gamma \rho = \alpha^2 A_1^2 V^{2(\alpha^2/f - 1)},
\tag{21}
\]

which further yields

\[
\rho = \frac{\alpha^2 A_1^2}{\left(2 \frac{\alpha^2}{f} - 1 + \gamma\right)} V^{2(\alpha^2/f - 1)}.
\tag{22}\]

From equation (20), we get

\[
p = \frac{\alpha^2 A_1^2 \gamma}{\left(2 \frac{\alpha^2}{f} - 1 + \gamma\right)} V^{2(\alpha^2/f - 1)}.
\tag{23}\]

Adding equations (8), 2 times (9) and 3 times equation (7), we get

\[
\left(\frac{\dot{a}_1}{a_1} + 2 \frac{\dot{a}_2}{a_2}\right) + 2 \left(\frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2^2}{a_2^2}\right) - \frac{6m_1^2}{a_1^2} = 12 \pi (\rho - p).
\tag{24}\]
From equation (13) we have
\[ \frac{\dot{V}}{V} = \left( \frac{\ddot{a}_1}{a_1} + \frac{2 \ddot{a}_2}{a_2} \right) + 2 \left( \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2^2}{a_2^2} \right). \] (25)

From equations (24), (25) and (20), we get
\[ \frac{\dot{V}}{V} - \frac{6m^2}{a_1^2} = 12\pi (1 - \gamma) \rho. \] (26)

Substituting equation (14) in equation (26), we get
\[ \frac{\dot{V}}{V} - \frac{6m^2}{V^{2/3}} = 12\pi (1 - \gamma) \rho. \] (27)

Substituting equation (22) in equation (27), we get
\[ \frac{\dot{V}}{V} - \frac{6m^2}{V^{2/3}} = 12\pi (1 - \gamma) \frac{\alpha^2 A_1^2}{\left( \frac{2\alpha^2}{f} - 1 + \gamma \right)} V^{2(\alpha^2/f - 1)}. \] (28)

This further gives
\[ \int \frac{dV}{\sqrt{\frac{12\pi (1 - \gamma) A_1^2 f}{2\alpha^2 - 1 + \gamma} V^{2\alpha^2/f} + 9m^2 V^{4/3} + k_1}} = t, \] (29)
where \( k_1 \) is integration constant.

For \( \gamma = 1 \) (Zeldovich fluid or stiff fluid) and \( k_1 = 0 \), the above equation gives
\[ V = m^3 t^3, \] (30)

Substituting equation (30) in equation (19), we get
\[ g = A_1 m^{3(\alpha^2/f - 1)} t^{3(\alpha^2/f - 1)}. \] (31)

Also, from equation \( \dot{C}(V) = g(V) \), we get the expression for creation field as
\[ C = \frac{A_1 m^{3(\alpha^2/f - 1)} t^{3(\alpha^2/f - 2)}}{\left( \frac{3\alpha^2}{f} - 2 \right)}. \] (32)

Substituting equation (30) in equation (22), the homogeneous mass density becomes
\[ \rho = \frac{1}{2} A_1^2 f m^{6(\alpha^2/f - 1)} t^{6(\alpha^2/f - 1)}. \] (33)
Using equation (23) and $\gamma = 1$, pressure becomes
\[
p = \frac{1}{2} A f m^6 (\alpha^2/f - 1)^6 (\alpha^2/f - 1). \tag{34}
\]
From equations (33) and (34), it is observed that for $f = \alpha^2$, there is no singularity in density and pressure.

Using equation (12), (14) and (30), we get
\[
a_1(t) = c^{2/3} m t, \tag{35}
a_2(t) = c^{-1/3} m t. \tag{36}
\]
Substituting the above values, the line element (5) reduces to
\[
ds^2 = dt^2 - c^{4/3} m^2 t^2 dx^2 - c^{-2/3} m^2 t^2 e^{-2m x} (dy^2 + dz^2). \tag{37}
\]

### 4 Physical Properties

We define $a = (a_1 a_2)^{1/3}$ as the average scale factor so that the Hubble’s parameter in our anisotropic models may be defined as
\[
H = \frac{\dot{a}}{a} = \frac{1}{3} \sum_{i=1}^{3} H_i,
\]
where $H_i = \frac{\dot{a}_i}{a_i}$ are directional Hubble’s factors in the direction of $x^i$, respectively.

The expansion scalar, $\theta$, is given by
\[
\theta = 3H, \quad \theta = \frac{3}{t}. \tag{38}
\]

The mean anisotropy parameter, $\bar{h}$, is given by
\[
\bar{h} = \frac{1}{3} \sum_{i=1}^{3} \left( \frac{\Delta H_i}{H} \right), \tag{39}
\]
\[
\bar{h} = 0.
\]

The shear scalar, $\sigma^2$, is given by
\[
\sigma^2 = \frac{1}{2} \left( \sum_{i=1}^{3} H_i^2 - 3H^2 \right) = \frac{3}{2} AH^2, \tag{40}
\]
\[
\sigma^2 = 0.
\]
The deceleration parameter \( q \) is given by
\[
q = \frac{d}{dt} \left( \frac{1}{H} \right) - 1,
\]
\( q = 0 \).

5 Conclusion

In this paper we have studied the space-time geometry corresponding to LRS Bianchi type-V in Hoyle-Narlikar creation field theory of gravitation [6–8]. The scale factors \( a_1 \) and \( a_2 \) are proportional to time \( t \). The deceleration parameter \( (q = 0) \) indicates that the universe is expanding with a constant velocity. The expansion scalar \( \theta \) starts with the infinite value at \( t = 0 \) and then gradually decreases. The expansion halts at \( t = \infty \). The mean anisotropy parameter is found to be zero. It means that the universe becomes isotropic throughout its evolution. Here we get the ratio \( \sigma/\theta = 0 \). Therefore, the model is isotropic. In general, we have observed that the creation field \( C \) is proportional to the time \( t \). That is, the creation of matter increases as time increases.

References