Neutrino Pair Bremsstrahlung in Supernova Environments

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Abstract. Core collapse Supernova is among the most energetic explosions in the Universe, releasing tremendous amounts of energy in the form of large number of neutrinos of all flavours. This is a result of a supernova shock wave formed when the iron core collapses gravitationally and rebounds at high density. There are strong indications that the neutron rich region between the resulting proto-neutron core and shock wave, referred to as the ‘hot bubble’ region is the site for rapid nucleosynthesis (r-process) of heavy nuclei, a key role played by supernovae. In this article we propose a possible reaction occurring between these neutrons and nuclei within the infalling matter from the region exterior to the stalled shock front. This reaction takes place when the collapse is well underway and leads to neutrino pair bremsstrahlung, in this ‘hot bubble’ region. The emissivity is calculated and the temperature dependence of this reaction is studied and compared with other dominant mechanisms of neutrino emission. It is found that this reaction compares favorably with other mechanisms, thereby suggesting that this reaction plays a significant role in the total neutrino luminosity, a critical factor for successful Supernova explosion. The emissivity of this process is comparable to electron-nucleus neutrino bremsstrahlung and nucleon-nucleon neutrino bremsstrahlung under certain conditions.

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1 Introduction

The study of supernova explosions has acquired immense importance in recent times [1–5] because of the knowledge they can impart on the formation of the Universe, as well as on neutrino flavour mixing. In particular, core collapse of Type II supernovae are spectacular explosions that mark the violent death of massive stars resulting in neutron stars or black holes. A tremendous amount of energy release of the order of $10^{53}$ ergs, at the extremely high rate of $10^{45–46}$ watts accompanies these explosions within the span of 1 second.
Massive stars, with masses greater than eight times the solar mass [1] evolve through hydrostatic burning to attain an ‘onion-skin’ structure with an inert iron core produced from the explosive burning of Silicon. The star begins to collapse when the core reaches Chandrasekhar mass. Gravitational work is done on the infalling matter resulting in increased temperature, density and elevated electron chemical potential. The latter begins to favor weak interaction conversion of protons to neutrons with the emission of electron neutrinos. Neutrino emission is the mechanism by which the star radiates energy and lepton number. The iron core becomes unstable due to the imbalance between the gravitational force tending to pull the stellar matter inwards and the outwardly directed neutron degeneracy force. This triggers the collapse mechanism presumed to characterize type II supernova (SN). The collapse proceeds to subnuclear densities where the stiff nuclear equation of state causes the core to bounce resulting in a stalled shock front.

As a result of the collapse of the iron core of a massive star to a neutron star, a hot proto-neutron star (PNS) is formed which radiates away its final binding energy as neutrinos. It is currently believed that the interaction of these neutrinos with the infalling matter is the mechanism responsible for exploding that part of the progenitor external to the neutron star and making a supernova [6]. When the explosion is well underway, the star consists of a proto-neutron core of approximately 80 km radius surrounded by another layer of nucleons extending to 100 km which is cooled by neutrinos. This is followed by a layer of neutrons and alpha particles extending to 200 km into the stalled shock front which is the heating region. This is surrounded by the infalling matter region which is iron rich, as depicted in Figure 1.

Although the specific physical conditions and particularly the astrophysical sites have not been identified unambiguously [7], it is generally believed that this ‘hot bubble’ region at the edge of the collapsing core of type II SN is the most
likely site for r-process nucleosynthesis [8]. The two main neutron capture processes for astrophysical nucleosynthesis are the slow (s) and rapid (r) n-capture processes [8]. Classification of these processes is based on the time scale for n-capture as compared to the time scale for the competing beta decay process. The hot bubble region formed as neutrinos escape [9] from the proto-neutron star during SN explosion is claimed to be a necessary and unavoidable aspect of successful SN explosion. The density of free neutrons is a key factor for explosive environments. Mayle and Wilson [10] have pointed out that the essential features of r-process nucleosynthesis include a large neutron to proton \( \frac{n}{p} \) ratio so that free neutrons are available for capture by iron group even after charged particle reactions have ceased. So r-processes can occur for \( \frac{n}{p} \approx 7 \) or 8 and proton mass fraction \( Y_e \sim 0.1 \). In fact, the entropy of the expanding matter and overall \( \frac{n}{p} \) ratio have been found to be more useful parameters in r-process than temperature and neutron density [11]. For times of the order of a few seconds, which greatly exceeds the 100s of millisecond required for collapse, the star gradually cools by emission of neutrinos of all flavors. As the neutrinos diffuse outwards they tend to remain in flavor equilibrium through reactions such as

\[
\nu_e + \bar{\nu}_e \leftrightarrow \nu_\mu + \bar{\nu}_\mu
\]

producing rough equipartition of flavor.

In this note we propose another process, namely, neutrino pair bremsstrahlung in neutron scattering from heavy nuclei. This process can occur simultaneously with the r-process, specially for low energy neutrons which do not undergo n-capture, due to the prevailing favorable conditions. It is during this epoch, when collapse is well underway, that neutrino pair bremsstrahlung contributes appreciably to neutrino luminosity, a critical issue in the successful completion of SN explosion. Neutrino pair bremsstrahlung in stellar environments has been extensively studied in stellar environments, including nucleon-nucleon interactions in neutron stars [12].

Thus, we consider the reaction

\[
n + \text{nucleus}(A, Z) \Rightarrow n + \text{nucleus}(A, Z) + \nu + \bar{\nu} \quad (1)
\]

during the above mentioned epoch in the hot bubble site of type II supernovae. The conditions when the reactions occur are high \( \frac{n}{p} \) ratio (or neutron density), high entropy (or temperature) and when the weak charged current reactions have ceased, so that the proton mass fraction, \( Y_e \) is low. The neutrinos interacting with the heavy nuclei such as iron, nickel, etc., from the infalling matter, are degenerate, having energies of the order of \( kT \), which are much less than the neutron Fermi energies. The neutrino emissivity for this process is calculated and compared with other similar processes.

The paper is organized as follows. Section 2 describes neutrino processes in type II Supernovae. In Section 3, the matrix element for the neutron-nucleus
weak neutral current interaction is presented, followed by emissivity calculations in Section 4. Concluding discussions are given in Section 5.

2 Neutrino Processes in Type II Supernova

Neutrino emission processes play an important role in the late stages of stellar evolution. Some of the dominant mechanisms of cooling of astrophysical objects include the photo-neutrino processes \((e^- + \gamma \rightarrow \nu \bar{\nu})\), pair annihilation \((e^- + e^+ \rightarrow \nu \bar{\nu})\), and plasma neutrino process \((\nu^* \rightarrow \nu \bar{\nu})\). Bremsstrahlung and Urca and modified Urca processes involving nucleons can be important in certain circumstances. Modified Urca process dominates the cooling of PNSs if direct Urca processes involving nucleons, hyperons or other strange particles do not occur.

In the Supernova environment, there are several crucial neutrino processes [13]. In the subnuclear density regime, the coherent scattering reaction

\[ \nu + (A, Z) \rightarrow \nu + (A, Z) \]

is the most important neutrino opacity source. The reactions

\[ \nu + e^- \rightarrow \nu + e^- \quad \nu + (A, Z) \rightarrow \nu + (A, Z)^* \]

are important in changing the neutrino energy and achieving thermodynamic equilibrium. The bremsstrahlung processes responsible for the production and thermalization of \(\mu\) and \(\tau\) neutrinos

\[ e^- + e^+ \rightarrow \nu \bar{\nu}, \quad (A, Z)^* \rightarrow (A, Z) + \nu \bar{\nu}, \quad (n, p) \rightarrow (n, p) + \nu \bar{\nu} \]

are dominated by nucleon bremsstrahlung for number densities \(> 0.005 \text{ fm}^{-3}\) and \(T < 15 \text{ MeV} \).

Neutrino bremsstrahlung from electron nucleus collisions proposed by Pontecorvo, Gandelman and Pinaev and later by Festa and Ruderman for electrons in the relativistic limit [14] is one of the major neutrino emission mechanisms occurring in the crust of stars and, therefore, referred as crust bremsstrahlung. The process considered is

\[ e^- + (A, Z) \rightarrow e^- + (A, Z) + \nu_x \bar{\nu}_x; \quad x = e, \mu, \tau. \]

More recently, Haensel et al. [15] computed the neutrino pair emissivity of relativistic degenerate electrons in a plasma of spherical nuclei as

\[ Q_{eA} = 3.229 \times 10^{17} Z Y_e \rho_{12} L T_9^6 \text{ erg cm}^{-3} \text{s}^{-1}, \]

\((L\) being a dimensionless constant, \(T_9 = T \times 10^{-9}\); \(\rho_{12} = \rho \times 10^{12} \text{ gms/cc})\) which is dominant in the high density limit.
Neutrino pair bremsstrahlung by nucleons in neutron star matter was studied by Flowers, Sutherland and Bond [12], namely, the processes

\[ n + n \rightarrow n + n + \nu \bar{\nu}; \quad n + p \rightarrow n + p + \nu \bar{\nu} \]

at nuclear matter densities and compared with the modified Urca process. Their estimates for the neutrino pair emissivity from the predominant axial vector component of the weak neutral current are, for the \( nn \) reaction and more interesting \( np \) reaction:

\[ \dot{\epsilon}^{A}_{nn} \approx 2 \times 10^{18} T_9^8 \text{ erg cm}^{-3} \text{s}^{-1}, \]
\[ \dot{\epsilon}^{A}_{np} \approx 10^{20} T_9^8 \text{ erg cm}^{-3} \text{s}^{-1}, \]

respectively, with \( T_9 \) in units of \( 10^9 \) K.

A more recent calculation by Sedrakian and Dieperink [16] for neutrino pair bremsstrahlung by baryons (B)

\[ B_1 + B_2 \rightarrow B_1 + B_2 + \nu \bar{\nu} \]

gives a numerical estimate of the neutrino emissivity as

\[ \epsilon_{\nu\nu} = (7.56 \times 10^{18} \text{ erg cm}^{-3} \text{s}^{-1}) T_7^2 I(T). \]

The triple integration \( I(T) \) yields a numerical value. On comparing their results with Friman and Maxwell [17] they found a strong suppression at relatively high temperatures and enhancement by a factor of 2 in the low temperature limit.

### 3 Matrix Element for Neutrino Pair Production

We consider the scattering of energetic neutrons of energy 100 MeV impinging on a heavy nucleus, like iron (\(^{56}\)Fe) and scattered with emission of neutrino pairs. The choice of 100 MeV neutrons is guided by the fact that in stars, the degenerate neutron fluid consists of neutrons having energies in this energy regime. At these energies, \( i.e. \) at energies much higher than thermal energy range 1 eV, the capture cross-section is negligible. As the nucleus is much heavier than the neutrons which are also in the non-relativistic energy range, momentum transfer to the nucleus is negligible. We consider a scalar interaction between the nucleus and neutrons, so \( \gamma \phi \) becomes \( \gamma \phi \).

Feynman diagrams for the process are shown in Figure 2. The initial and final four momenta of the neutrons are \((q, E)\) and \((q', E')\), respectively. Neutrons are incident on a fixed nucleus interact via a vector nuclear potential \( V_\mu \), whose momentum space transform is \( \phi_\mu \) and are subsequently scattered with the emission of neutrino pairs of four momenta \( k_1 \) and \( k_2 \). The nucleons in the neutron star matter are very degenerate and therefore \( \nu \bar{\nu} \) pairs that are radiated have very low
energies which are much less than nucleon Fermi energies. This gives rise to soft $\nu\pi$ bremsstrahlung propagator and a radiation matrix element. The matrix element is a sum of two terms due to gauge invariance (see Figure 2). Each term is the product of the four fermion interaction given by

$$\langle u_\nu(k_2) | \gamma^\mu (1 - \gamma^5) | u_\nu(k_1) \rangle,$$

(2)

a weak neutral current coupling term and a neutral potential coupling term ($\gamma\phi$). The above neutrino current term is for each of the three flavours $\nu_e$, $\nu\mu$, and $\nu_\tau$, so a sum is to be performed over the three flavours.

The interaction of the neutron-heavy nucleus is of the form

$$\epsilon^\mu \gamma^\mu c_v - c_a \gamma^5 \epsilon^\mu \gamma^\mu c_v + \epsilon^\mu \gamma^\mu c_v - c_a \gamma^5 \epsilon^\mu \gamma^\mu c_v - c_a \gamma^5 \epsilon^\mu \gamma^\mu c_v - c_a \gamma^5$$

(3)

with $\epsilon^\mu$ being the neutron polarization, $c_v$, and $c_a$ being the vector and axial vector coupling constants, respectively, and

$$G_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{m^2}.\quad(4)$$

Since the neutrons (mass $m$) are degenerate, with energies much less than Fermi energies, the second term on the right hand side of Eq. (4), is negligible and we assume that the neutrons are on the mass shell. Calculations are performed in the non-relativistic limit with momentum transfer to the nucleus being negligible. With these considerations, the amplitude for the reaction may be expressed as a product of three factors: the T-matrix for the scattering of the nucleons on the mass shell, a nucleon propagator and a neutrino radiation matrix element. The matrix element is a sum of two terms due to gauge invariance.
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The matrix element for the process is thus

\[
M = -\frac{ie^2G}{\sqrt{2}}(u_{\nu}(k_2)|\gamma^\mu(1 - \gamma_5)|u_{\nu}(k_1)) \\
\times \langle u_{u'}(q')|(e_v - e_a\gamma_5)\frac{\gamma_\phi}{\gamma q - m} + (e_v - e_a\gamma_5)\frac{\gamma_\phi}{\gamma q' - m}|u_{u'}(q)\rangle, \tag{5}
\]

where the us' represent the spinors, γs the Dirac matrices and \(e_v\) and \(e_a\) are the vector and axial vector constants for weak neutral currents, and G is the Fermi coupling constant \((G \approx 10^{-5}/m_p^2, m_p\) being the proton mass). In order to compute the matrix element, the following substitutions are made: \(q_1 = q' + X\) and \(q_2 = q - X\). Here \(q_1\) is the four-momentum of the neutron after energy transfer \((k)\) to the nucleus and before neutrino emission and \(q_2\) is the four-momentum of the neutron before energy transfer and after neutrino emission, in the two Feynmann diagrams of Figure 1, and \(X = k_1 + k_2\). Due to degeneracy, \(X^2 \ll qX\) and so the approximation to the internal fermion propagators become

\[
\frac{1}{\gamma q_1 - m} = \frac{q' + m}{2q'X} \tag{6}
\]

and

\[
\frac{1}{\gamma q_2 - m} = -\frac{q + m}{2qX} \tag{7}
\]

and

\[
\frac{1}{(qX)(q'X)} = \frac{1}{(qX)^2} \tag{8}
\]

in the non-relativistic limit.

In the low temperature limit at which the interaction is considered, the only important neutron states are those near the Fermi surfaces. The total neutrino momentum \(|q|^2\) is of the order \(kT\) and can be safely neglected (integration over unobserved neutrino momenta \([18]\) is carried out) in the momentum conserving \(\delta\) function and the remaining neutron momenta can be put on their respective Fermi surfaces. The square of the matrix element summed over the final spin polarizations of the neutron and averaged over the initial neutron polarization and simplified is

\[
\frac{1}{2} |M|^2 = \frac{G^2e^4}{2} (R_1 + R_2). \tag{9}
\]

This matrix element squared is multiplied by a nuclear form factor \(F(q)\) and

\[
R_1 = 32(C_v^2 - C_a^2)\phi_0^2\left[\frac{2|m^2 - (qq')|}{(qX)^2}(X_0^2 + X^2) - m^2\left(2 - \frac{qX}{q'X} - \frac{q'X}{qX}\right) - \frac{m^2}{(qX)^2}\left(2X_0(qq' - m^2)\right)\right]
\]

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and

\[ R_2 = 32(C_v^2 + C_a^2) \phi_0^2 \left[ \frac{X^2(qy')}{(qX)^2} \left( [(qy') - 2m^2] + m^2 \left[ 2 - \frac{qX}{q'y} \right] \frac{q'X}{qX} \right] + \frac{m^2}{(qX)^2} \left[ 2X_0^2((qy') - m^2X_0^2) \right] \right] \] (10)

with \( \phi_0 \) being the value of the nuclear potential. The form factor \( F(q) \) (with \( q \) being a function of momentum and incident angle), considering a spherical atomic nucleus with an uniform proton core of radius \( R_c \), is given by

\[ F(q) = \frac{3}{(qR_c)^3} [\sin(qR_c) - qR_c \cos(qR_c)] \]

which is an adequate description of nuclear densities \( \rho < 10^{13} \text{ gm cm}^{-3} \).

The leading order terms proportional to the vector term \( C_v^2 \) cancel out and, therefore, the contribution from this term is much smaller than the axial vector term. Further, the ratio of the vector to axial vector term is inversely proportional to the square of the nucleon mass and thus vanishes in the limit of infinite nucleon mass as required by vector current conservation.

In addition, spin-orbit interactions affect the axial vector current and the three spin projection components have been incorporated in the numerical estimate of emissivity given in a later Section.

4 Emissivity

In astrophysical objects processes like Eq. (1) occur, for instance, in massive stars as well as neutron stars which result from the compactification of the core of a massive star after a supernova explosion. According to current belief, nucleosynthesis of heavy elements takes place in the ‘hot bubble’ region when the collapse is well underway, at temperatures of around \( 10^7 \text{ K} \). This nucleosynthesis occurs via thermonuclear reactions to generate the fusion of nucleons with nuclei to produce heavier nuclei. This fusion chain continues till \( ^{56}\text{Fe} \) is produced, after which the fusion processes cease to be exothermic. Typical Supernova explosions yield about \( 0.7M_\odot \) of \( ^{56}\text{Fe} \) nuclei. The core of these objects is surrounded by a sea of relativistic degenerate electron gas whose density is such that the whole system is electrically neutral. When the Fermi energy of the electrons is greater than the proton-neutron mass difference, inverse beta decay occurs and electrons are captured by the nuclei to produce progressively rich nuclei. The region thus consists of nuclei in a degenerate fluid of neutrons and permeated by a sea of degenerate, relativistic electron gas and the rapid neutron capture is a possible nuclear reaction for heavier nuclei production. Simultaneously, reactions such as Eq. (1) discussed in this paper, occur for neutrons
which have low energy and are not captured, but scattered. At typical neutron star and other massive star densities, the average neutron Fermi energy is around 100 MeV, which is non-relativistic and degenerates for temperatures less than $10^{12}$ K.

We thus consider positively charged heavy nuclei like iron in a non-relativistic degenerate neutron fluid. These neutrons have a Fermi distribution given by

$$f_i = \frac{1}{1 + \exp[(E_i - \mu)/T]}$$

(11)

for $i = 1, 2$. Here $T$ is the temperature and $\mu$ is the chemical potential of the neutrons. The neutrinos are free and, therefore, do not have a Fermi distribution function. Further, we consider the interaction between the nucleus and neutrons is given by a Wood-Saxon potential of the form

$$V(r) = -(V + iW)g(r) + V_{so} \frac{1}{r} \frac{dg}{dr} (\vec{\sigma} \cdot \vec{L})$$

where

$$g(r) = \left(1 + \exp\left(\frac{r - R}{a}\right)\right)^{-1}$$

and $V, W, V_{so}$ are constants of the order of several tens of MeV.

The emissivity rate is given by

$$dE/dt = 3 \int \prod_{i=1}^{3} \frac{dp_i}{(2\pi)^3} f_1(1 - f_2) \int dX X (2\pi)^4 \delta^4(q' + X - q)$$

$$\times \int \int \frac{d^3k_1}{2k_1} \frac{d^3k_2}{2k_2} \delta^4(k_1 + k_2 - k)|M|^2.$$  \hspace{1cm} (12)

Here $(p_i = q, q', X)$. The above equation includes the presence of the distribution function $f_1$ of the incident neutrons and $f_2$ of the scattered neutrons. The usual change of variables is made, namely $E = \mu + T x_1, E' = \mu - T x_2$, and $X = T x_3$ to get

$$dE/dt = G^2 m^2 / (2 \omega^2 + 3e_0^2) \phi_0^2 / (q_n T_n) T^5 \int dx_3 x_3 \int_0^{\infty} dx_1 \int_{(m-\mu)/T}^{(\mu-m)/T} dx_2$$

$$I(x_1, x_2, x_3) \delta(x_1 - x_2 - x_3) / [(1 + \exp(x_1))(1 + \exp(x_2))]$$

(13)

where $I(x_1, x_2, x_3)$ is the angular integral which is described in the previous Section. This above integral is to be performed numerically, in the general case. For the range of temperatures considered, $(\mu - m)/T \gg 10$ and so the limits of integration of $x_1$ are $(-\infty, \infty)$ and of $x_3$ are $(\infty, -\infty)$. After simplifying

$$dE/dt \propto T^5 \int_0^{\infty} dx_3 x_3 (x_3^2 + \pi^2) / (1 + \exp(x_3)) I(x_1, x_2, x_3),$$

(14)
which is to be integrated numerically. The estimate for emissivity is $7.67 \times 10^{17} \frac{\phi_0^2}{q_n T_n} \rho_{12} Y_n T_n^5$ erg cm$^{-3}$s$^{-1}$, at subnuclear matter density (subscript $n$ denoting the C.M. momentum and energy) which is an order of magnitude lower than the neutrino luminosity in core collapse supernova of type II quoted in [19–21] but compares favourably with the reactions considered in [22], thus suggesting that neutrino pair bremsstrahlung can be a process contributing to the total luminosity.

Thus, we conclude that, the emissivity rate is proportional to the fifth power of temperature ($\propto T^5$) and therefore produces appreciable cooling. Also, this mechanism is significant during the condensation process when nuclei are being formed. Further, upon comparing the neutrino luminosity of this process with the total neutrino luminosity, it appears that other processes [19–22], we find that this reaction also contributes significantly to the total value of luminosity, thus suggesting that this process too, can play a significant role in producing a successful Supernova explosion, which is critically dependent on neutrino luminosity.

5 Discussions

The emissivity rates for the electron nucleus bremsstrahlung process [15] as well as our results are plotted in Figure 3 for iron with a nuclear density of $10^{10}$ g cm$^{-3}$ in the temperature range $10^6 < T < 10^{10}$ K.

It was found that, the nucleon nucleus process dominates at lower temperatures when the neutrons are abundant and electron ratio is low, whereas, electron processes dominate at high temperature and high electron fraction $Y_e$, which are typical conditions in the neutron star crust. We have examined the temperature dependence of emissivity of the neutrino-antineutrino pair and found that it varies as the fifth power of temperature, compared to other stellar processes which are proportional to higher powers of temperature (e.g. modified URCA process [23]) This cooling is thus more effective during the condensation phases when nuclei are being formed. It may be concluded that this neutrino pair production mechanism might play a significant role in the successful completion of Supernova explosion.

As discussed in an earlier Section, electron-nucleus neutrino pair bremsstrahlung and nucleon-nucleon neutrino pair bremsstrahlung are some of the dominant bremsstrahlung processes occurring during advanced stages of stellar evolution. The former process occurs mainly in the crust whereas the latter occurs in the core. Under certain conditions described in the Introduction (high temperature/entropy, low proton mass fraction, realistic abundance of iron and other heavy nuclei, sub-nuclear densities between $4.3 \times 10^{11} < \rho \leq 4 \times 10^{14}$ g cm$^{-3}$ etc.) and temperatures less than $T < 10^{10}$ K neutrino emissivity rate from
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Figure 3. Emissivity vs. Temperature (solid line: neutron-nucleus bremsstrahlung; dashed line: electron-nucleus bremsstrahlung).

the process described in this paper (viz., neutron-nucleus) occurring in the core of the Supernova is comparable with the electron-nucleus neutrino emissivity occurring in the crust region and an order of magnitude lower than the neutrino emissivity from the nucleon-nucleon process in the core. (Similar processes also occur in neutron stars [24].) Thus this process can contribute significantly to the total neutrino luminosity.

References

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