Beam-Based Measurement of Dynamical Characteristics in Nuclotron

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Abstract. The real values of dynamical characteristics in Nuclotron have been measured by means of both sinusoidal and kick excitations of the beam. For measurement of betatron tunes the method based on the Beam Transfer Function has been used. Beam-based modeling by means of Orbit Response Matrix allowed us to calculate the amplitude beta function and the betatron phase advance with big accuracy. Applying this method, one can also calibrate the Beam Position Monitors, and reveal the bad-working units.

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1 Introduction

In first approximation an accelerator is described by the linear optical model. To measure the dynamical characteristics of this model is among the major tasks of accelerator commissioning and optimum exploitation. The real accelerator optics usually differs from the machine design. Knowing the correct values of the accelerator optical functions, such as beta function, betatron tune, and betatron phase advance is of major importance.

For measurement of the accelerator optical functions a system of Beam Position Monitors (BPM) and orbit corrector magnets (CM) is used. BPMs measure the beam center of charge position with respect to the reference orbit. CMs produce dipole kicks that distort the closed orbit.

The behavior of any dynamical system is fully described by its response to one of the standard excitations. The response of a dynamical system to unit step function is known as transient function. The response of a dynamical system to a Dirac pulse function is called impulse transient function or sometimes weight function. The response of a dynamical system to sinusoidal excitation is the well-known frequency characteristic or transfer function.
Each of these characteristics describe both transient and steady-state modes.

In accelerator practice the transfer function is known as Beam Transfer Function (BTF). The shock beam excitation by means of a fast kicker magnet results in the coherent beam oscillations. The response of the center of charge positions to kicks in closed orbit correctors is called Orbit Response Matrix (ORM). A general theorem states that the transfer function is a Fourier transfer of the impulse response.

One of the most important parameters of any cyclic accelerator is the tune of the betatron oscillations – \( Q \). The value of \( Q \) determines the position of the so-called working point of the accelerator. The working point must be kept away from any dangerous resonances during the whole accelerator cycle. Many accelerator parameters, such as chromaticity and the dependence of the tune on the amplitude are firmly connected with the \( Q \)-value.

One may say that the efficiency of the accelerator and the possibility to reach the maximum beam intensity are determined by the proper choice and maintenance of the betatron tune.

On the other hand the structural beta function describes the beam envelope and the betatron phase advance.

For measurement of the correct values of the structural beta functions and of betatron phase advance in Nuclotron, a method based on the Orbit Response Matrix has been used.

## 2 Betatron Tune Measurement in Nuclotron

Nowadays, all the existing synchrotrons, storage rings and colliders have electronic systems for on-line tune measurement.

The Nuclotron tune measurement system is based on the exciting of transverse oscillations of the beam applying an external sinusoidal signal and measurement of the corresponding center of charge response. This response is known as Beam Transfer Function (BTF).

Transverse BTF [1] is determined by the ratio of the amplitude of the center of charge oscillations to the external sinusoidal excitation

\[
\frac{r(\omega)}{\hat{F}} = r_r(\omega) + i r_i(\omega) \tag{1}
\]

where \( \hat{F} \) is the amplitude of the external sinusoidal excitation \( F = \hat{F} \cos \omega_t t \), and \( \langle y \rangle \) is the corresponding center of charge deviation.

The external signal excites slow waves with a frequency:

\[
\omega_{\beta S} = (n - Q)\omega_0, \tag{2}
\]
and fast waves with a frequency:

\[ \omega_{\beta f} = (n + Q)\omega_0, \]  

(3)

\( n \) being any integer.

In accelerator practice we are interested not in the individual oscillations of the beam particles, but in the center of charge motion. This is the only measured by BPMs value. For this reason, we have to average the individual particle responses over the distribution of the betatron frequency \( \omega_{\beta} \).

It can be shown that

\[ \langle y \rangle_{\text{fast}} = \frac{\hat{F}}{2\omega_e} (r_r \cos(\omega_e t) + r_i \sin(\omega_e t)) \]  

(4)

where \( \omega_e \) is the excitation frequency.

The resistive part of the BTF is

\[ r_r = P.V. \int \frac{f(\omega_{\beta f})d\omega_{\beta f}}{\omega_e - \omega_{\beta f}} \]  

(5)

where \( f(\omega_{\beta}) \) is the betatron frequency distribution function and \( P.V. \) denotes the Cauchy principal value of the integral.

The reactive part of the BTF for fast waves is

\[ r_i = -\pi f(\omega_e). \]  

(6)

For slow waves one must substitute \( \omega_{\beta f} \) with \( \omega_{\beta s} \).

A simplified block-diagram of the used equipment is shown in Figure 1 [2].

For the excitation of the transverse betatron oscillations in NUCLOTRON we have used a broadband amplifier. It consists of 9 identical blocks with a transformer output. One of these blocks is used as a preamplifier. Its output signal excites in parallel the other 8 blocks. The secondary windings of these blocks are connected in series and this gives 250 V on the load in the frequency range 0.6–6.0 MHz. The load represents 100 pF capacity in parallel with a 75 Ω resistor. The maximum output power is 0.6 KW. Input and control signals are transmitted by 0.5 km matched cables to and from the accelerator ring.

The already existing pick-ups used for orbit measurements have been used also for the beam excitation and for the response measurement. These pick-ups are 11 cm long and are situated in special boxes in the vacuum chamber of the accelerator. The minimum voltage on the pick-up, which we can still reliably measure, is 200 \( \mu \)V. This corresponds to 1 V on the ADC input.

A spectrum of simultaneous measurement of horizontal and vertical betatron oscillations is shown in Figure 2.
For betatron tune measurement in Nuclotron, we tried also the method with observation of damped coherent beam oscillations.

The successive beam centroid positions in a given pickup electrode are measured and LSQ fitted to the theoretical curve. For a beam with Gaussian distribution

![Diagram of the tune measurement equipment](image)

**Figure 1.** Simplified block-diagram of the tune measurement equipment.

![Spectrum of simultaneous measurement of horizontal and vertical betatron oscillations](image)

**Figure 2.** Spectrum of simultaneous measurement of horizontal and vertical betatron oscillations in Nuclotron (Along the horizontal axis the channel number of the spectrum analyzer is shown; the vertical axis is in relative units.).
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Figure 3. Coherent beam oscillations. Horizontal scale is proportional to the time elapsed after injection in relative units; 2000 divisions corresponds to 200 particle revolutions.

of the betatron tunes, this theoretical curve reads

\[ \langle y_n \rangle = A \exp \left( -\frac{(n2\pi\sigma Q)^2}{2} \right) \cos(n2\pi Q + \phi) + y_{co} \]  

(7)

where \( \langle y_n \rangle \) are the turn-by-turn center of charge deviations in the chosen BPM and \( y_{co} \) is the closed orbit at the monitor’s azimuth.

In this method the successive beam center of charge deviations in a given BPM is measured, and the collected data are fitted by the theoretical curve (7). From the fitting the fractional part of the betatron tune and the tune r.m.s. could be determined.

The coherent oscillations of the beam center of charge are shown in Figure 3. The beating is due to small coupling between horizontal and vertical oscillations. The Fourier spectrum of the coherent betatron oscillations consists of betatron sidebands at frequencies \( (n \pm Q)f_0 \), \( n \) being any integer and \( f_0 \) – the revolution frequency. The measured Fourier spectrum of vertical coherent betatron oscillations in Nuclotron is shown in Figure 4. The double pick at \( (1 - q)f_0 \) is due to the coupling.

These two methods can reveal only the fractional part ‘q’ of the betatron tune \( Q = n + q \).

The integer part ‘n’ of the tune can be determined from the spectrum of closed orbit. The following relation between the Fourier harmonics of the random perturbations in the magnetic fields and in the quadrupole positions – \( \tilde{f}_k \), \( g_k \) and of the corresponding closed orbit distortions – \( u_k \), \( v_k \) holds [1]:

\[ u_k = \frac{Q^2}{Q^2 - k^2} \tilde{f}_k \quad v_k = \frac{Q^2}{Q^2 - k^2} g_k \]  

(8)

As the spectrum of the perturbations is white, the maximum of the orbit spectrum drops on a frequency number ‘k’ , which is the closest to the \( Q \) value. This
Figure 4. Fourier spectrum of vertical coherent betatron oscillations in Nuclotron (the revolution frequency for ions with $Z/A = 1/2$ is $f_0 = 0.122$ MHz, the fractional vertical betatron tune is $q = 0.414$).

is illustrated by Figure 5, in which the Nuclotron closed orbit and its Fourier spectrum are shown. It is clearly seen that the integer part of the betatron tune in the Nuclotron is equal to seven.

Summarizing all the measurements one can conclude that the horizontal betatron tune in Nuclotron is equal to $Q_h = 7.463$, while the vertical is $Q_v = 7.414$.

Figure 5. Measured Nuclotron closed orbit and orbit spectrum: (A) — horizontal closed orbit; (B) — vertical closed orbit; (C) — horizontal orbit spectrum; (D) — vertical orbit spectrum.
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These values differ from the design values, which are: $Q_h = 6.80$ and $Q_v = 6.85$. After discussions it was concluded that the reason for this discrepancy lies in the interaction between magnetic fluxes in some neighbouring structural elements. The experimental verification of this hypothesis is in the way.

Knowing the correct values of the betatron tunes is crucial for the optimization of the accelerator operation and for the intensity increase. It is very important and for the proper work of the injection and extraction systems.

3 Measurement of Optical Functions in Nuclotron

One powerful method for measurement of optical functions is based on the Orbit Response Matrix (ORM). The ORM is defined by [3]

$$R_{ij} = \frac{dx_i}{d\varepsilon_j}$$  \hspace{1cm} (9)

where $d x_i$ is the change of the center of charge position $x_i$ at the $i$-th BPM due to a change $d \varepsilon_j$ in the kick at $j$-th CM.

One should distinguish between the theoretical ORM – $R_{ij}^{\text{theor}}$, which reflects the properties of the accepted linear optical model of the accelerator, and the measured ORM – $R_{ij}^{\text{meas}}$.

We could expand the theoretical ORM to the first order in the errors in quadrupole strengths $(\Delta k_l)$

$$R_{ij}^{\text{theor}} = R_{ij0} + \sum_l \frac{\partial R_{ij}}{\partial k_l} \Delta k_l.$$  \hspace{1cm} (10)

We should fit the measured ORM – $R_{ij}^{\text{meas}}$ with $g_i R_{ij}^{\text{theor}} s_j$, $g_i$ being the BPMs gain factors and $s_j$ being the correctors scale factors.

In the fit we should take into account that a kick $\Delta \varepsilon_j$ in the $j$-th CM leads to a change of the synchronous particle energy and thus it introduces an additional momentum error:

$$\frac{dp}{p} = \frac{1}{E_0^2 - \alpha} \frac{D_j \varepsilon_j}{L}$$  \hspace{1cm} (11)

provided the dispersion at the corrector $D_j$ is non-vanishing. In (11) $\alpha$ denotes the momentum compaction factor, $L$ is the machine circumference, $E$, $E_0$ are the total and the rest energies of the particles and $\gamma$ is the relativistic factor.

For that reason the theoretical response matrix becomes:

$$R_{ij}^{\text{theor}} = \sqrt{\frac{\beta_i \beta_j}{2 \sin \pi Q}} \cos(\pi Q - |\mu_i - \mu_j|) + \frac{D_i D_j}{L \eta}$$  \hspace{1cm} (12)
where $\eta = (1/\gamma^2 - \alpha)$ and $\mu$ is the phase advance.

Because there is no direct way to measure the dispersion at the corrector magnet, $(\Delta p/p)_j$ should be considered as a fit parameter.

There is one more thing that should be taken into account, namely that the different BPMs have different noise level. This can be measured for each BPM by successive orbit measurements under constant corrector strengths. Let $\sigma_i$ be the rms orbit deviations for the $i$-th BPM. This rms deviation is a measure for the noise level associated with this BPM. Hence we must introduce the weights $(1/\sigma_i)$ in the fitting.

Summarizing we reach to the following system of equations for $g_i$, $s_j$, $\Delta k_i$, and $(dp/p)_j$:

$$R_{ij}^{\text{meas}} = \frac{1}{\sigma_i} g_i (R_{ij0} + \sum_l \frac{\partial R_{ij}}{\partial k_l} \Delta k_l + \frac{D_i D_j}{L \eta}) s_j$$ (13)

As (13) includes more equations than the unknowns, it should be solved in LSQ.
sense. The best way to do this is the singular value decomposition.

The orbit response matrix is a good instrument for studying machine optics due to the very large number of experimental points. This allows for the right individual gradients to be determined. Using the calibrated accelerator model it is possible to compute the right values of the beta functions.

The orbit response matrix gives direct information about the bad operating BPMs and the broken symmetry of the ring.

The experimental setup is shown in Figure 6 [4].

By now we have measured eight columns of BRM – 160 numbers as a whole. This allowed us to reveal the real values of the amplitude $\beta$-functions, although with less accuracy.

The design and measured $\beta$-functions are shown in Figure 7. The discrepancy is rather small, less than five percents.

To check the proper work of the BPMs, we have realized the following experiment. First of all, we measured the closed orbit produced by the random perturbations in the magnetic fields and to the random displacement of quadrupoles. After that we fired one of the orbit correctors, increasing the excitation current from 1 to 2 and 3 Amps, and again measured the closed orbit. Subtracting from these latter orbits the initial orbit we received the closed orbit due only to the corrector. The difference orbits in horizontal plane are shown on Figure 8. It is clearly seen that some problems with the BPMs occur around the azimuths 153 m (BPM-#12) and 185 m (BPM-#15).
Figure 8. Horizontal closed orbits due to excitement of corrector HK74 with currents 1, 2, and 3 Amps. Some problems with the proper work of BPMs #12 and #15.

4 Conclusions

The frequencies of betatron oscillations in the superconducting heavy ion synchrotron Nuclotron of the Laboratory for High Energies of the Joint Institute for Nuclear Research in Dubna have been measured by means of both Beam Transfer Function and Coherent Beam Oscillations methods. The necessary electronic equipment and data processing software have been developed. These kinds of measurements are nowadays available from the control room of the accelerator and can be applied to any point of the accelerator cycle.

The elements of the Beam Response Matrix have been partially measured. This allowed us to reveal the real values of the amplitude $\beta$ – functions of the accelerator.

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