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INCOMPATIBILITY OF KINK SPACE–TIME IN SCALE COVARIANT THEORY OF GRAVITY

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Abstract. In this paper, we have taken an attempt to study the feasibility of scale covariant theory (Canuto et al, 1977) of gravity in kink space-time with a gauge function β being independent of time and a matter field in the form of a perfect fluid. It is found that the perfect fluid does not survive and kink space–time turns out to be flat in this theory.

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1. Introduction

In alternative theories of gravitation, scalar tensor theories proposed by Brans and Dicke [1], Nordtvedt [4], Sen and Dunn [2], Ross [3], and Barber [5] are most important among them. In the theory proposed by Brans and Dicke [1] there exists a variable gravitational parameter G. Another theory, which admits a variable G, is the scale covariant theory of Canuto et al [6]. The scale covariant theory is also available as an alternative to general relativity (Wesson [7], Will [8]). Within the framework of the scale covariant theory, the cosmological constant L appears as a variable parameter. On the other hand, Einstein's general relativity does not admit the possibility of variable G.

In this theory, Reddy and Venkateswarlu [9] have shown the existence of Birkhoff's theorem of general relativity, when the gauge function is independent of time. In this paper, we have taken an attempt to construct the cosmological model of kink space-time with a matter field in the form of a perfect fluid in the scale covariant theory proposed by Canuto et al [6], when the gauge function is independent of time.

In scale invariant theory of gravity Mohanty and Mishra [10, 11] have already studied the incompatibility of non-diagonal and diagonal Bianchi type II and III space-times respectively with a matter field in the form of a perfect fluid. However, in these papers they have shown that this theory is not feasible for both Bianchi type II and III metrics.

2. Field Equations

The field equations in the scale covariant theory are [6]

$$R_{ij} - \frac{1}{2}Rg_{ij} + f_{ij} = -\kappa T_{ij} + \Lambda g_{ij} \tag{1}$$

where

$$\beta f_{ij} = 2\beta\beta_{;ij} - 4\beta_{,i}\beta_{,j} - (g^{ab}\beta_{,a}\beta_{,b} - 2g^{ab}\beta_{;ab}) \tag{2}$$

in which β is a scalar function (the gauge function) satisfying $0 < \beta < \infty$. In these equations R_{ij} is Ricci tensor; R is Ricci scalar; g_{ij} is the metric tensor; Λ is the cosmological constant; $\kappa = \dfrac{8\pi G}{c^4}$, where κ is the gravitational parameter; and T_{ij} is the energy momentum tensor. A semicolon denotes covariant differentiation whereas a comma denotes partial differentiation.

Here we consider a kink space–time in the form

$$\mathrm{d}s^2 = -\cos 2\alpha\,\mathrm{d}t^2 - 2\sin 2\alpha\,\mathrm{d}r\,\mathrm{d}t + \cos 2\alpha\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega \tag{3}$$

where $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \cos^2\theta\,\mathrm{d}\varphi$ and $\alpha = \alpha(r)$.

The energy momentum tensor for perfect fluid distribution is given by

$$T_{ij} = (p + \rho)U_iU_j + pg_{ij} \tag{4}$$

together with

$$g_{ij}U^iU^j = -1 \tag{5}$$

where U^i is the four velocity vector of the fluid, and p and ρ are the proper isotropic pressure and energy density respectively.

By use of comoving coordinates $(0,0,0,\sqrt{\sec 2\alpha})$, the field Eqs (1) for the metric (3) can be written as:

$$\begin{gathered}\frac{2}{r^2}\sin^2\alpha+\frac{2}{r}\sin 2\alpha\alpha_1-2\sin 2\alpha\alpha_1(1+2\sin^2 2\alpha)\frac{\beta_1}{\beta}+\cos 2\alpha\frac{\beta_1^2}{\beta^2}\\ -2\cos 2\alpha\frac{\beta_{11}}{\beta}+\frac{4}{r}\cos 2\alpha\frac{\beta_1}{\beta}+\sec 2\alpha\left(2\frac{\beta_{11}}{\beta}-4\frac{\beta_1^2}{\beta^2}\right)-\Lambda=-\kappa p\end{gathered}\tag{6}$$

$$\begin{gathered}\sin 2\alpha\alpha_{11}+\frac{2}{r}\sin 2\alpha\alpha_1+2\cos 2\alpha\alpha_1^2-4\sin^3 2\alpha\alpha_1\frac{\beta_1}{\beta}\\ +\cos 2\alpha\frac{\beta_1^2}{\beta^2}-2\cos 2\alpha\frac{\beta_{11}}{\beta}+\frac{2}{r}\cos 2\alpha\frac{\beta_1}{\beta}-\Lambda=-\kappa p\end{gathered}\tag{7}$$

$$\begin{gathered}\frac{2}{r^2}\sin^2\alpha+\frac{2}{r}\sin 2\alpha\alpha_1+2\sin 2\alpha\alpha_1(1-2\sin^2 2\alpha)\frac{\beta_1}{\beta}\\ +\cos 2\alpha\frac{\beta_1^2}{\beta^2}-2\cos 2\alpha\frac{\beta_{11}}{\beta}+\frac{4}{r}\cos 2\alpha\frac{\beta_1}{\beta}-\Lambda=\kappa(p+\rho)\sec^2 2\alpha-\kappa p\end{gathered}\tag{8}$$

$$\begin{gathered}\frac{2}{r^2}\sin^2\alpha+\frac{2}{r}\sin 2\alpha\alpha_1+2\sin 2\alpha\alpha_1(1-2\sin^2 2\alpha)\frac{\beta_1}{\beta}\\ +\cos 2\alpha\frac{\beta_1^2}{\beta^2}-2\cos 2\alpha\frac{\beta_{11}}{\beta}+\frac{4}{r}\cos 2\alpha\frac{\beta_1}{\beta}-\Lambda=-\kappa p\,.\end{gathered}\tag{9}$$

Equations (8) and (9) yields

$$p+\rho=0$$

Now Eqns (6)–(9) reduces to three equations as

$$\begin{gathered}\frac{2}{r^2}\sin^2\alpha+\frac{2}{r}\sin 2\alpha\alpha_1-2\sin 2\alpha\alpha_1(1+2\sin^2 2\alpha)\frac{\beta_1}{\beta}+\cos 2\alpha\frac{\beta_1^2}{\beta^2}\\ -2\cos 2\alpha\frac{\beta_{11}}{\beta}+\frac{4}{r}\cos 2\alpha\frac{\beta_1}{\beta}+\sec 2\alpha\left(2\frac{\beta_{11}}{\beta}-4\frac{\beta_1^2}{\beta^2}\right)-\Lambda=-\kappa p\end{gathered}\tag{10}$$

$$\begin{gathered}\sin 2\alpha\alpha_{11}+\frac{2}{r}\sin 2\alpha\alpha_1+2\cos 2\alpha\alpha_1^2-4\sin^3 2\alpha\alpha_1\frac{\beta_1}{\beta}\\ +\cos 2\alpha\frac{\beta_1^2}{\beta^2}-2\cos 2\alpha\frac{\beta_{11}}{\beta}+\frac{2}{r}\cos 2\alpha\frac{\beta_1}{\beta}-\Lambda=-\kappa p\end{gathered}\tag{11}$$

$$\begin{gathered}\frac{2}{r^2}\sin^2\alpha+\frac{2}{r}\sin 2\alpha\alpha_1+2\sin 2\alpha\alpha_1(1-2\sin^2 2\alpha)\frac{\beta_1}{\beta}\\ +\cos 2\alpha\frac{\beta_1^2}{\beta^2}-2\cos 2\alpha\frac{\beta_{11}}{\beta}+\frac{4}{r}\cos 2\alpha\frac{\beta_1}{\beta}-\Lambda=-\kappa p\,.\end{gathered}\tag{12}$$

Due to the non-linear nature of the field Eqs (10)–(12), we assume

$$\beta = \frac{1}{ar+b} \tag{13}$$

where $a(\neq 0)$ and b are real constants.

Now, the set of field Eqs (10)–(12) become

$$\begin{aligned}\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin 2\alpha\alpha_1 + \frac{2a}{ar+b}\sin 2\alpha\alpha_1 + \frac{4a}{ar+b}\sin^3 2\alpha\alpha_1 \\ + \frac{3a^2}{(ar+b)^2}\cos 2\alpha + \frac{4a}{r(ar+b)}\cos 2\alpha - \Lambda = -\kappa p\end{aligned} \tag{14}$$

$$\begin{aligned}\sin 2\alpha\alpha_{11} + \frac{2}{r}\sin 2\alpha\alpha_1 + 2\cos 2\alpha\alpha_1^2 - \frac{2a}{ar+b}\cos 2\alpha\alpha_1 \\ - \frac{4a}{ar+b}\sin^3 2\alpha\alpha_1 - \frac{3a^2}{(ar+b)^2}\cos 2\alpha - \Lambda = -\kappa p\end{aligned} \tag{15}$$

$$\begin{aligned}\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin 2\alpha\alpha_1 - \frac{2a}{ar+b}\sin 2\alpha\alpha_1 + \frac{4a}{ar+b}\sin^3 2\alpha\alpha_1 \\ - \frac{3a^2}{(ar+b)^2}\cos 2\alpha - \frac{4a}{r(ar+b)}\cos 2\alpha - \Lambda = -\kappa p\,.\end{aligned} \tag{16}$$

Eqns (14) and (16) yields

$$\alpha = \text{const}\ .$$

3. Discussions

Canuto et al [6] derived the generalized gravitational field equations in three different ways: (a) by performing a direct scale transformation; (b) by extending Riemannian geometry to Weyl geometry, through the introduction of the notion of cotensors; and (c) from a variational principle. Here we have assumed that (a) the metric is non-diagonal and kink type; (ii) the gauge function $\beta = \dfrac{1}{ar+b}$ where $a(\neq 0)$ and b are real constants; and (c) the energy momentum tensor is that of a perfect fluid. Since the metric of the space–time is homogeneous, it is quite natural to consider here Dirac gauge function. However it is found that the space-time reduces to Minkowskian and the matter field does not survive in scale covariant theory.

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