DEPARTURE FROM THERMAL EQUILIBRIUM IN THE CONTRACTION REGION OF HIGH-PRESSURE Hg ARCS

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Abstract. This paper describes the thermal equilibrium behavior in the contraction region of a cylindrical high-pressure discharge lamp. Using the energy conservation equations for heavy and light (electrons) plasma components, a bifluid model for walls stabilized mercury arc is made.

1. Introduction.
Calculation of the discharge parameters in different mathematical models is based on various approximations. The most used approximations in the high-pressure mercury arc discharge models are: one in which a positive plasma column with an infinite stretch is considered (the electrodes effects are neglected) and the second one in which the plasma is considered to be close to the local thermodynamic equilibrium (LTE).

Indirect measurement of the $6^3P_2$, $6^3P_1$, and $6^3P_0$ populations show that these are different from the same levels populations calculated using Boltzmann's law [1]. In the last years, more and more authors deny the LTE hypothesis [2].

The aim of this paper is to describe the thermal equilibrium behavior in the contraction region of a cylindrical high-pressure discharge lamp.

2. Model.
The arc is composed from two components: the heavy particles (neutrals and positive ions) and the electrons distributed by Maxwell function at $T_h$ and $T_e$ temperature respectively. The energy conservation in a stationary regime for the heavy and light components expressed in terms of cylindrical coordinates $(r,z)$ can be written as follows:

- for the atoms and ions:
  \[ \frac{1}{r} \frac{\partial}{\partial r} \left( r k_p \frac{\partial T_h}{\partial r} \right) + \frac{\partial}{\partial z} \left( k_p \frac{\partial T_h}{\partial z} \right) + E_{el} = 0 \]  
  \( (1) \)

- for the electrons:
  \[ \frac{1}{r} \frac{\partial}{\partial r} \left( r k_e \frac{\partial T_e}{\partial r} \right) + \frac{\partial}{\partial z} \left( k_e \frac{\partial T_e}{\partial z} \right) + jE - U_{rad} - E_{el} = 0 \]  
  \( (2) \)

where $k_p$ and $k_e$ are the plasma and electron conductivity, $j$ and $E$ are respectively the current density and the electric field, $U_{rad}$ is the radiation term and $E_{el}$ represent the elastic collision.
The electric field intensity is calculated by Ohm's law: 
\[ I = 2\pi E \sigma r dr \], where \( I \) is the discharge current, \( \sigma \) is the electric conductivity, \( R \) the tube radius and \( e \) is the electron charge. 
All transport coefficients are calculated using the first approximation of the gas kinetic theory developed by Hirschfelder et al. [3]. For the radiant power I preferred the Elenbaas formula [4]: 
\[ U_{\text{rad}} = 1.08 \times 10^{-10} n_{Hg} \langle \nu \rangle \exp\left( -eV^* / k_B T_e \right) \], where \( n_{Hg} \) is the mercury atoms density and \( V^* = 7.8 \text{ V} \).

The energy exchange rate \( \dot{E}_{\text{el}} \) between electrons and heavy particles can be calculated as:
\[ \dot{E}_{\text{el}} = 3 \frac{m_e}{M} (1 - T_h / T_e) n_e (T_e) \langle \nu_e \rangle k_B T_e \], where \( m_e \) is the electron mass, \( M \) is the mercury mass, \( n_e \) is the electrons density and \( \langle \nu_e \rangle \) is the mean elastic collision frequency between electrons and heavy particles. It can be written as \( \langle \nu_e \rangle = \left( \nu_{e}^{th} \right) / \langle \lambda \rangle \), where \( \left( \nu_{e}^{th} \right) \) is the average thermal speed and \( \langle \lambda \rangle = 1 / \langle Q \rangle n_{Hg} \) is the mean free path, with \( \langle Q \rangle = 1.2 \times 10^{-18} \text{ m}^2 \).

3. Boundary conditions for \( T_e \) and \( T_s \)
For the heavy temperature \( T_h \) we have used the same Dirichlet - Neumann mixed boundary conditions as in reference [5]. Regarding the electron temperature distribution \( T_e \), Chen and Pfender [6] have shown that the influence of the boundary conditions is not significant. Consequently, I considered a constant value \( T_e = 1000 \) K on the all frontier.

The discharge tube used in modeling is schematically presented in Fig. 1, the geometrical dimensions being: \( R = 10 \text{ mm} \), \( l = 10 \text{ mm} \), \( L = 92 \text{ mm} \) and \( r_e = 1 \text{ mm} \).

4. Results and discussions.
The partial differential equations (1) and (2) were solved using the Finite Element Method, with triangular grid and variable step. The code has been checked for numerical diffusion effects. Few results are presented as follows.
In Fig. 2 are presented the light and heavy plasma components temperature distributions obtained as solutions of eqs. (1) and (2).

![Figure 2](image)

**Fig. 2.** Electronic temperature (a) and heavy particles temperature (b) in contraction region for \( I=3.6 \) A and \( p=1.5 \) atm.

Using Saha and Dalton laws, I have obtained the electrons and mercury atoms distributions. These distributions are presented in Fig. 3.

![Figure 3](image)

**Fig. 3.** Electrons and mercury atoms densities (in m\(^{-3}\)) distributions in the contraction zone of the lamp at \( I=3.6 \) A and \( p=1.5 \) atm.

The difference between electronic fluid temperature and heavy plasma component temperature \( T_e - T_h \) represent a departure from LTE. This dependence is shown in Fig. 4, for different discharge pressure or different discharge current. As is shown in Fig. 4a), this difference diminishes when the work pressure increase. At \( I=3.6 \) A, for \( p>1 \) atm., the plasma is close to the thermodynamic equilibrium. The significant temperature difference variation in the cathode vicinity is due to different thermal gradient. The electronic temperature longitudinal gradient is high while the heavy temperature gradient increase more slowly in front of the cathode.

Fig. 4 b) show that for a constant discharge pressure, the departure from LTE became zero for the high discharge current. At low current values, the departure from LTE is more
pronounced. In this case, the electron density is lower and the number of electrons/atoms collisions diminish. The collision processes cannot realize the plasma thermalisation.

![Graph](image)

**Fig. 4.** Dependence of the LTE deviation vs distance in front of the electrodes

The expansion zone is a region in which the current channel expands from the electrode diameter (in diffuse mode operation) to that of the positive plasma column. Starting from the electric plasma conductivity \(\sigma = n_e e \mu_e\), I have calculated the distance from the central discharge axis to the point where the conductivity decrease "e" times from the maximum value. This distance represents the discharge channel radius \(r_c\). In **Fig. 5** is presented the channel radius dependence on the distance measured in front of the electrode, at I=3.6 A and p=1.5 respectively 3 atm.

![Graph](image)

**Fig. 5** Discharge channel radius dependence on the distance in front of the electrode

**References**


