DIFFRACTION CHARACTERISTICS OF AN APERTURE WITH COSINE PEARL FORM

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A new type of diffraction object with form of an aperture limited between two successive intersections of two conjugated cosine curves is proposed. The wave and intensity distribution in process of Fraunhofer diffraction for this aperture is defined.

The investigation is needed for knowing the so-called form-factor of diffractograms, if this type of aperture is taken as structural element of the grating or of the initiator of a fractal object.

Standard forms of two-dimensional apertures, treated in the process of Fraunhofer diffraction, are those bounded by a circle, rectangle [1], triangle and sextangle [2].

Here we present the results of our investigations concerning a new type of aperture bounded by two conjugate cosine function curves (Fig.1), in the interval between their two successive intersections with apside axis.

Its transmission or aperture function is defined by:

$$A(\xi, \eta) = \begin{cases} 1 & \frac{\pi}{2}\xi \leq \eta \leq a \cos \frac{\pi}{2}\xi \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (1)

In (1) \(a\) is the amplitude of the cosine curves, \(\alpha\) is their special frequency.

The diffraction screen with aperture together with a planar convex lens, whose transmission function is

$$T(\xi, \eta) = \exp \left[ i k \left( \frac{\xi^2 + \eta^2}{2f} \right) \right]$$  \hspace{1cm} (2)

With \(f\) being the lens focal distance, and \(k=2\pi/\lambda\) is the wave number for the used light of wavelength \(\lambda\) are located on position \(z=\xi\). Being illuminated by a plane wave \(\Psi_0 = C_0 \exp(-ik\xi)\), the lens aperture system diffracts the light in the focal plane \(E\) of the lens situated at distance \(x=\xi+f\) (Fig. 2).

Denoting by \(x\) and \(y\) the coordinates in the observation plane \(E\), we use the
Fresnel diffraction integral to define the diffracted wave distribution \(\Psi(x,y)\)

\[
\Psi(x,y) = \frac{i k C_0 e^{-i k z}}{2\pi(z-c)} \int \int \exp\left(-i k \left[\frac{z}{2(z-c)} \left(\xi - x\right)^2 + \left(\eta - y\right)^2\right]\right) \exp(i \omega_x \xi) \exp(i \omega_y \eta) \, d\xi \, d\eta.
\]

(3)

Taking into account the definitions (1) and (2) we find for the wave field (3) the expression

\[
\Psi(\omega_x, \omega_y) = C \int \frac{\sin(\omega_x \xi) \left[\exp(i \omega_x \xi) \exp(i \omega_y \eta) \right] d\xi}{\omega_x}
\]

where we used the notations:

\[
C = \frac{i k C_0}{2\pi f} \exp\left(-i k \left[\frac{z}{2f} \left(x^2 + y^2\right)\right]\right)
\]

(5)

and

\[
\omega_x = \frac{kx}{f}, \quad \omega_y = \frac{ky}{f}
\]

(6)

The integration over the coordinate \(\eta\) is trivial and results into

\[
\Psi(\omega_x, \omega_y) = \frac{2C}{\omega_y} \int \frac{\sin(\omega_x \xi) \sin[\omega_x \cos(\alpha \xi)]}{\omega_x} d\xi.
\]

(7)

To perform the integration over the second variable we use the identity

\[
\sin(\omega_x \cos(\alpha \xi)) = \sum_{m=0}^{\infty} (-1)^m J_{2m+1}(\omega_x) \cos((2m+1)\xi)
\]

(8)

known from the theory of Bessel functions [3]. The final result of the integration is:

\[
\Psi(\omega_x, \omega_y) = \frac{\pi C}{\alpha} \sum_{m=0}^{\infty} (-1)^m \frac{J_{2m+1}(\omega_x)}{2m+1} \left[\sin\left\{\frac{\pi}{2} \left(\frac{\omega_y}{\alpha} + (2m+1)\right)\right\} + \sin\left\{\frac{\pi}{2} \left(\frac{\omega_y}{\alpha} - (2m+1)\right)\right\}\right]
\]

(9)

Such wave distribution dictates the diffracted irradiance distribution of the form

\[
I(\omega_x, \omega_y) = |\Psi(\omega_x, \omega_y)|^2 = \pi^2 \left|\frac{\alpha}{\omega_x}\right|^2 \left[\sum_{m=0}^{\infty} (-1)^m \frac{J_{2m+1}(\omega_x)}{2m+1} \left[\sin\left\{\frac{\pi}{2} \left(\frac{\omega_y}{\alpha} + (2m+1)\right)\right\} + \sin\left\{\frac{\pi}{2} \left(\frac{\omega_y}{\alpha} - (2m+1)\right)\right\}\right]\right]^2
\]

(10)

It is easily seen that it has a central symmetry i.e.
\[
I(\omega_x, \omega_y) = I(-\omega_x, \omega_y) = I(\omega_x, -\omega_y) = I(-\omega_x, -\omega_y)
\] (11)

Considering that the aperture surface is \(P^2 - 4(\alpha/\alpha)\), and the irradiance at the root of the frequency plane is \(I(0, 0) = 16|C|^2 \left( \frac{\alpha}{\alpha} \right)^2 = P^2 |C|^2\), we have for the irradiance distribution the following expression

\[
I(\omega_x, \omega_y) = I(0, 0) \left( \frac{\pi}{4} \right)^2 \left[ \sum_{m=0}^{\infty} \left( \frac{\omega_y}{\alpha} \right)^{2m+1} \left( J_{2m+1}(2\omega_y) J_{2m}(\omega_y) + J_{2m}(2\omega_y) J_{2m+1}(\omega_y) \right) \right]
\] (12)

It was programmed for the case of an aperture with unity amplitude and frequency (\(\alpha = 1; \alpha = 1\)).

![Fig. 3a](image)

![Fig. 3b](image)

![Fig. 3c](image)
On the Fig 3. we give the lines of equally irradiance in the screen plane (a) and three-dimensional graph of the distribution (12) (b).

We also made the diffractiongrams for apertures with \(\alpha = 1; \alpha = 1\) (Fig. 4a), \(\alpha = 1; \alpha = 1/2\) (Fig. 4b), \(\alpha = 1/2; \alpha = 1\) (Fig. 4c) and \(\alpha = 1/4; \alpha = 1\) (Fig. 4d).

![Figures 4a to 4d](image_url)

The comparison of the graph (fig. 3) and the diffractiongram given on the Fig. 4a shows an excellent agreement of the theoretical and the experimental result.

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**References**